3D Bézier Volume Model From a Stick Figure Using Semi-Simploidal Sets

D. Aholou, S. Peltier *(XLIM/ASALI, Univ. Poitiers)*

G. Morin *(IRIT, Univ. Toulouse)*

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Nancy – virtuel

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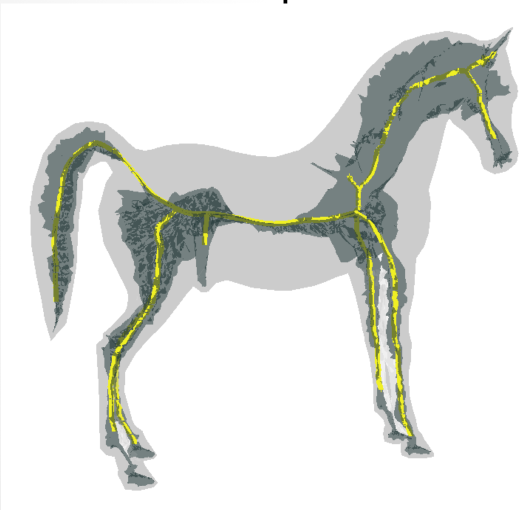
Context & Goal

Modeling a 3D free-form object

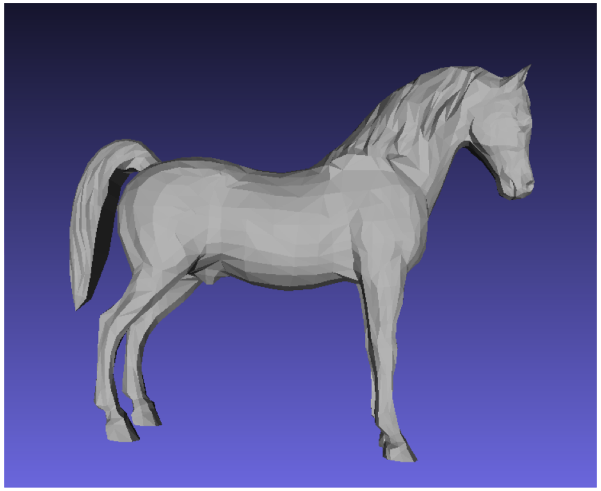
Skeletons

+ intuitive modeling
+ adapted for



Surface mesh

+ classical model



Volume mesh

+ supports physical
+ simulation
+ necessary for 3D printing

courtesy Gmsh

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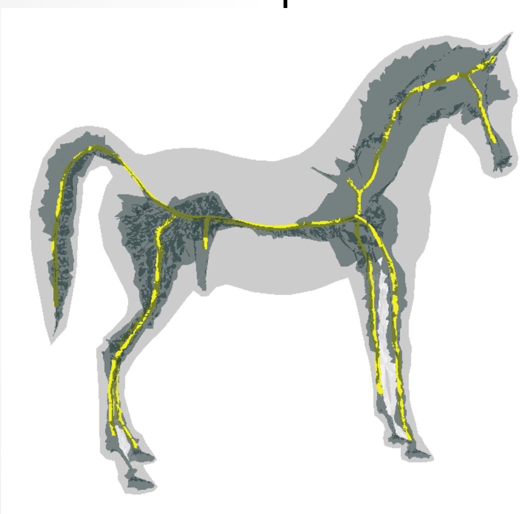

Surface mesh

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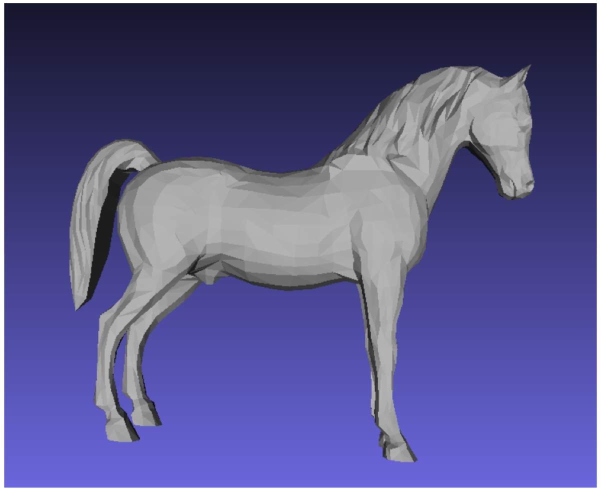

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courtesy Gmsh

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Context & Goal

Goals :

Benefit from advantages of the three representations :

- start from a (1D) **skeleton** in $\mathbb{R}^3$
- a **surface (quad) mesh** around the skeleton
  - **same** number of quads around each edge
  - **small** number of quads around each edge
- a **volume mesh** filling the surface mesh
  - only **one 3D cell type**
  - **non degenerate cells**

+ Control topology and geometry

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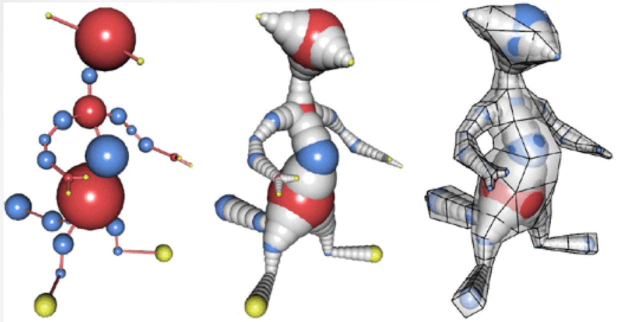
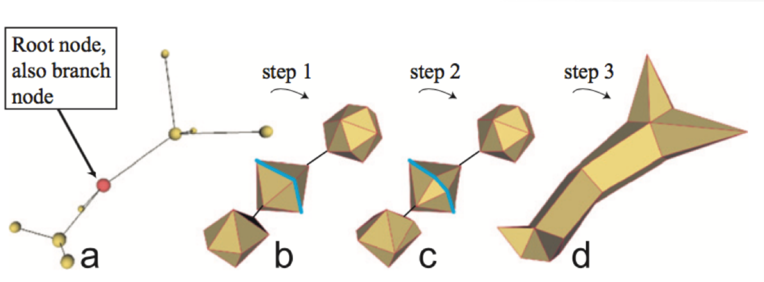
Mesh Scaffold from Skeleton

- **B-Mesh**

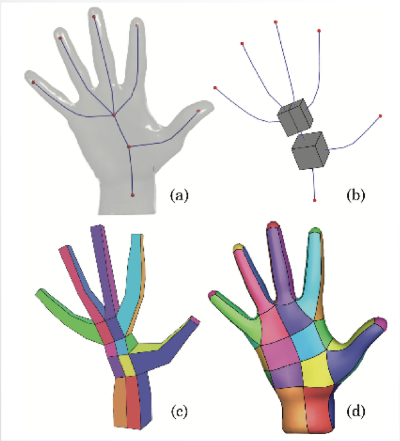


B-Mesh, Z Ji et al. - CFG, 2010

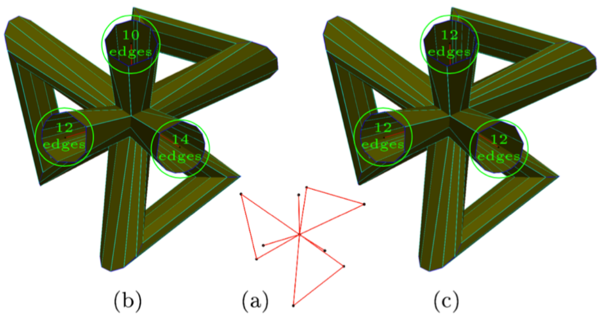


Converting skeletal structures to quad dominant meshes, Bærentzen et al. SMI 2012

- **Quad layout**



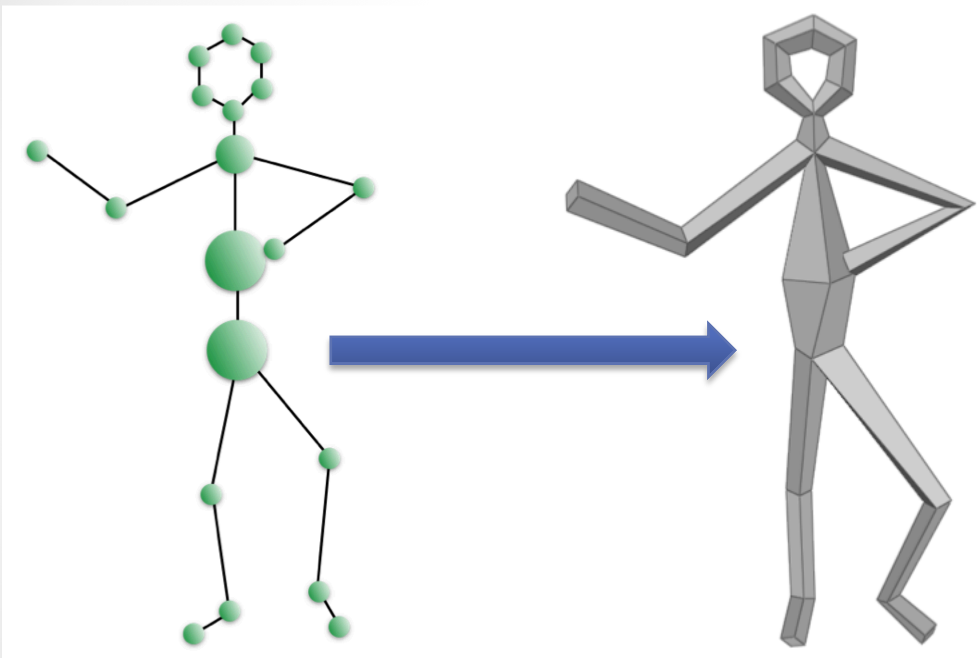
Extraction of the Quad Layout of a Triangle Mesh Guided by Its Curve Skeleton, Usai et al. ACM ToG 2015



Scaffolding skeletons using spherical Voronoi diagrams: Feasibility, regularity and symmetry, Suárez et al. – CAD 2018

Mesh Scaffold from Skeleton

Scaffolding a Skeleton, Panotopoulou et al., Research in Shape Analysis, 2018

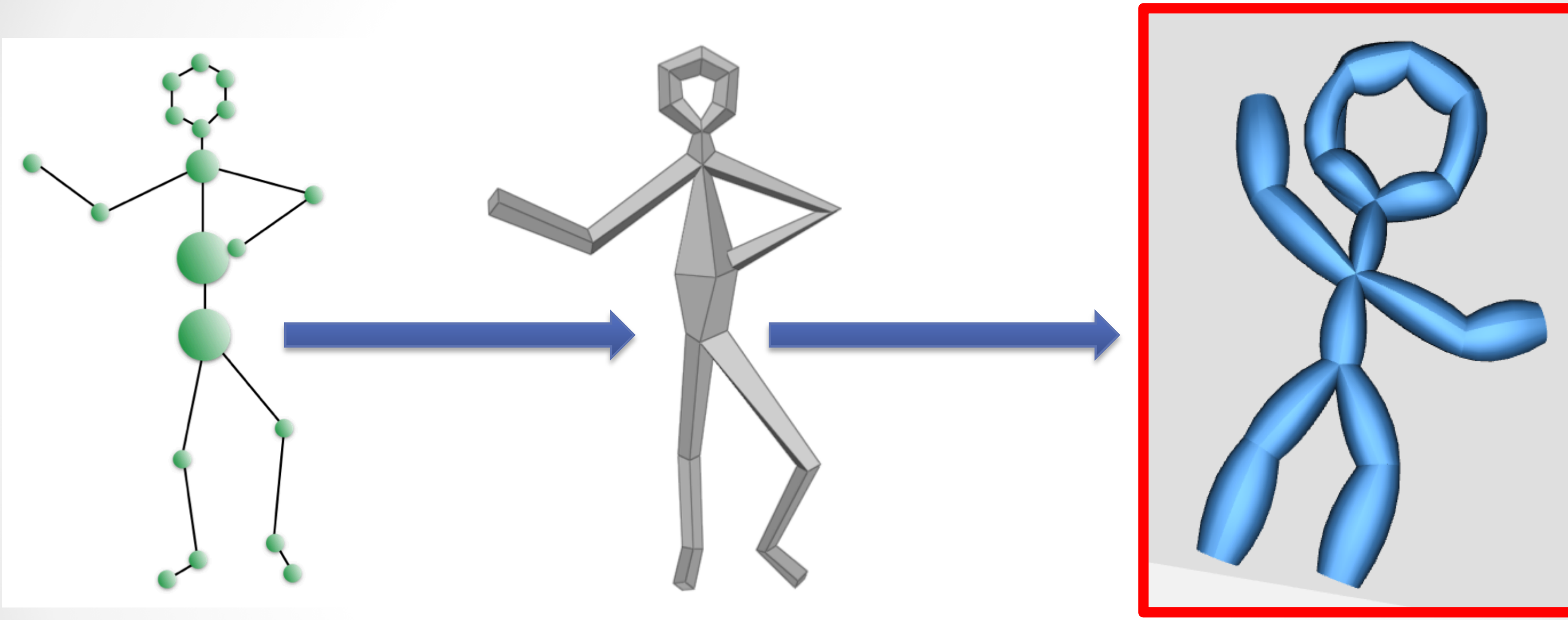


Skeleton **Quad mesh**

4 quads around each edge
-> minimal for regular branches (Euler formula)

Volume model from Skeleton

Scaffolding a Skeleton, Panotopoulou et al., Research in Shape Analysis, 2018

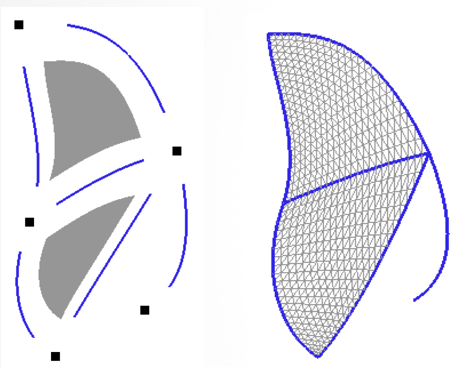


Skeleton **Quad mesh**

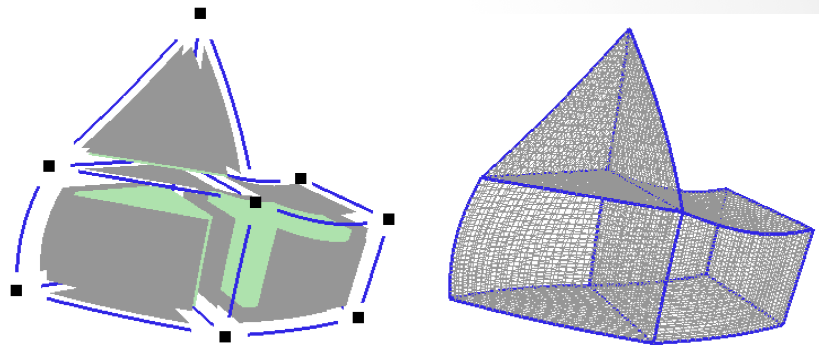
+ volume model
+ non linear

Based on Simploidal Bézier sets

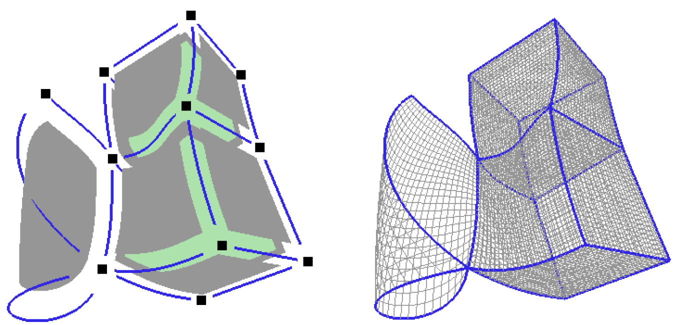
- **Handling Subdivided objects**
  - Combinatorial structure **(topology)** **+ operations**
  - Bézier Embedding **(geometry)**



simplicial



simploidal



cubical

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Outline

- ✓ Context & Goal
- Filling in the quad mesh
- Handling non linear, free form 3D objects
- Joining the branches in 3D
- Conclusion – Future Work

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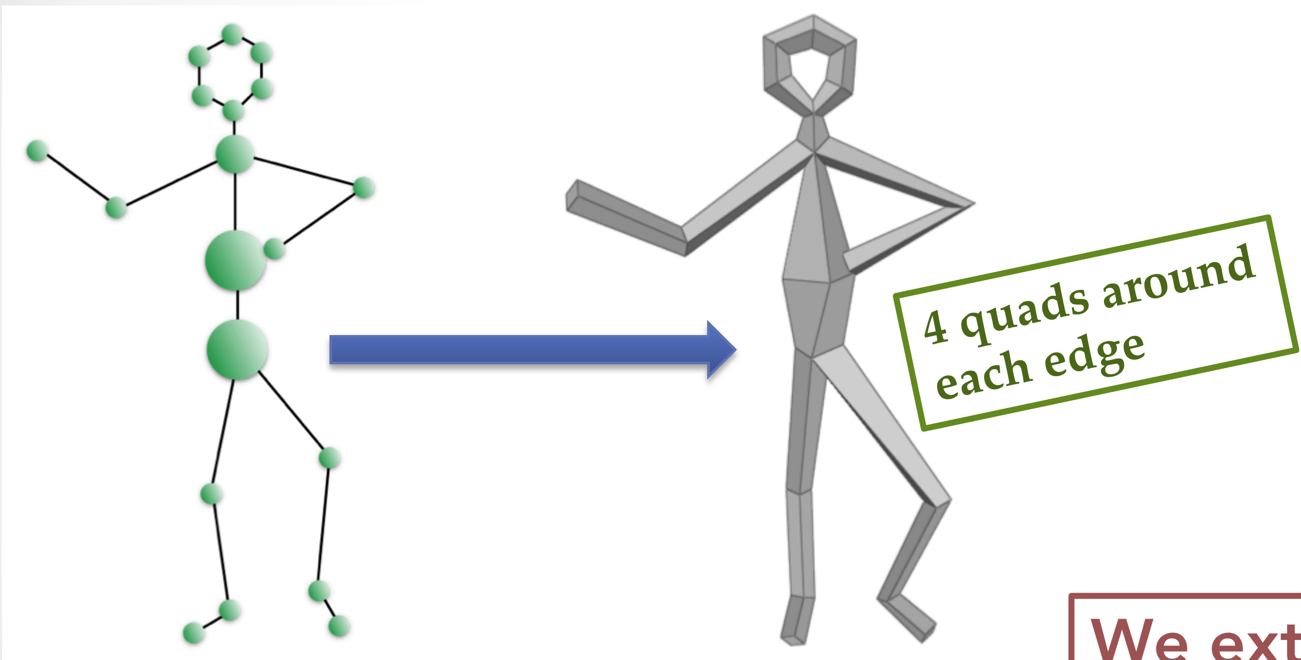
Mesh Scaffold from Skeleton

Scaffolding a Skeleton, Panotopoulou et al., Research in Shape Analysis, 2018



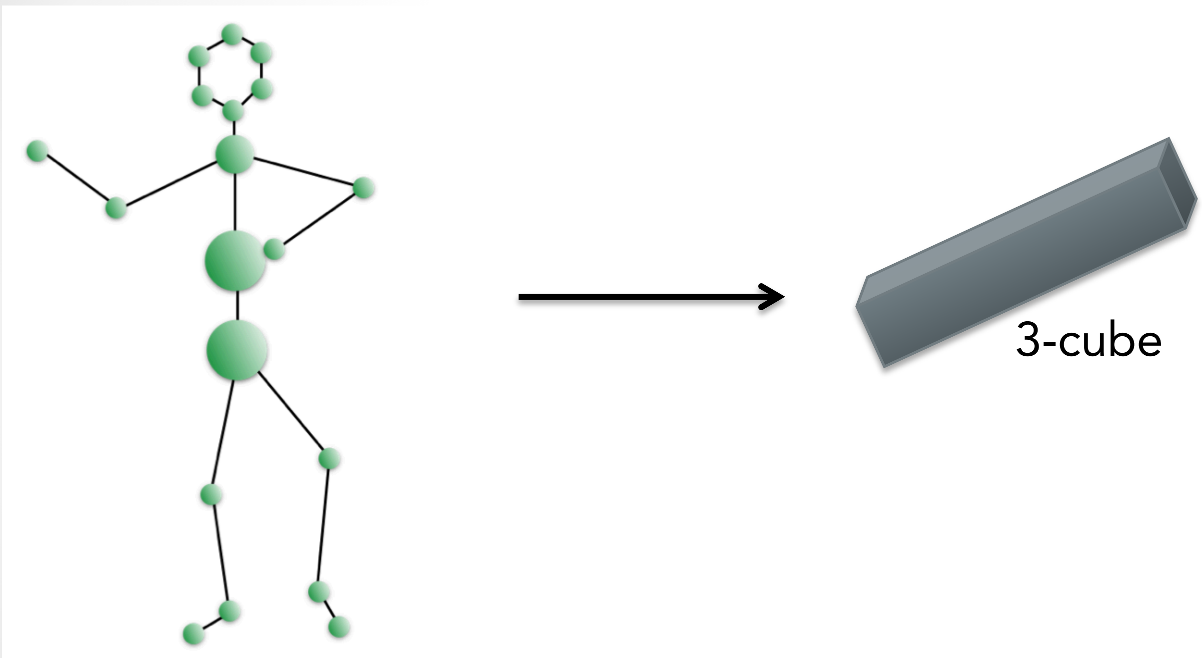
Skeleton

Quad mesh

We extend this approach to volume mesh

Filling The Scaffold

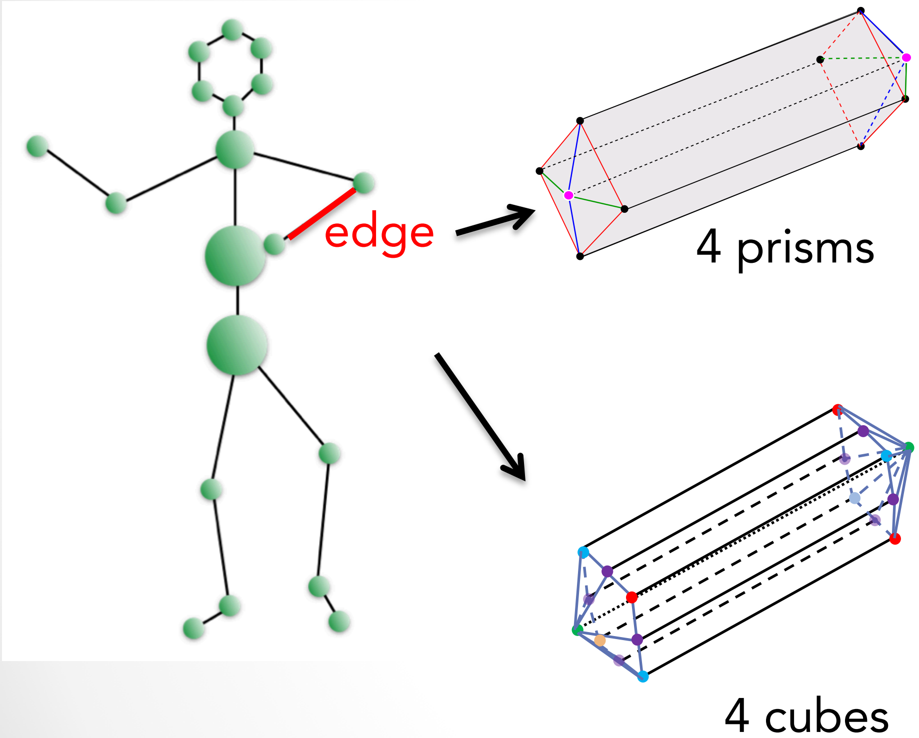
First Idea: cubical sets



Filling the nodes ?

Filling The Scaffold

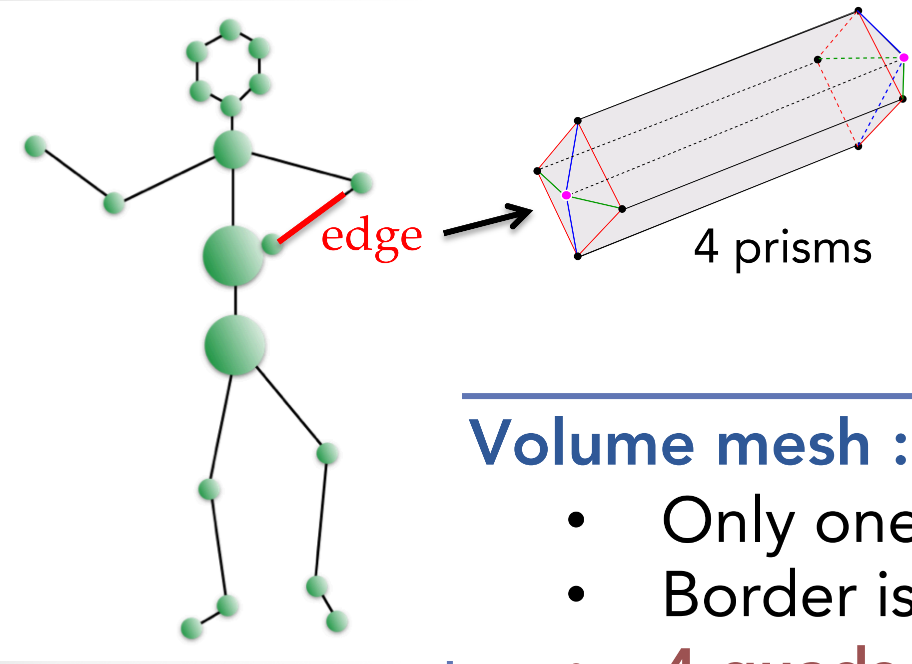
simploidal sets



Presented by Paul Viville,
GTMG 2020

Filling The Scaffold

simploidal sets

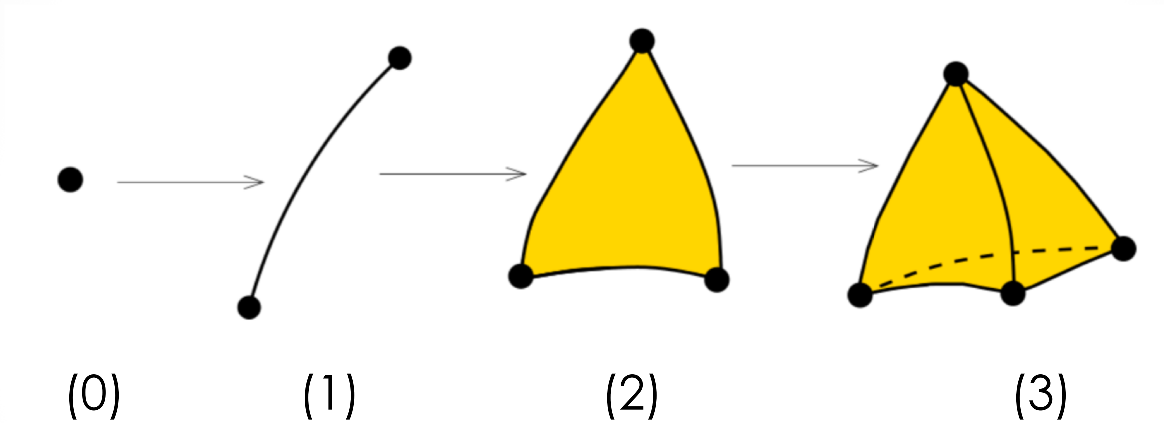


Volume mesh :

- Only one cell type : **prisms**
- Border is a **surface mesh**
- **4 quads** around each edge
- **Topology**
- **Bézier embedding**

Simplices

- Generation of simplices



Cone operation

type : (k)

dimension : k

Cuboids

- Generation of cuboids

Product operation

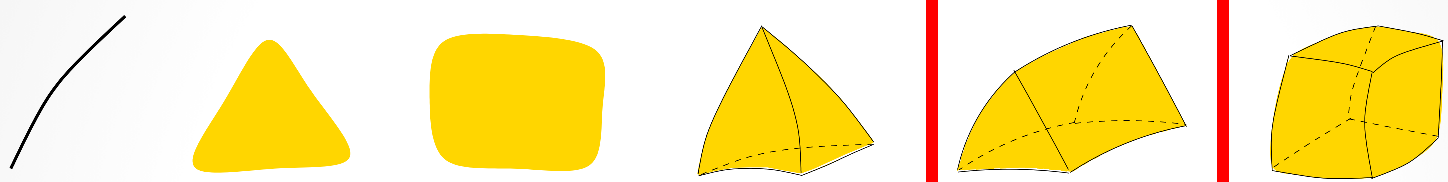
type : (1, … , 1)

dimension : k

Simploids

[Dahmen - Michelli 82]

- Products of simplices



(1) (2) (1,1) (3) (2,1) (1,1,1)

- **type** : $(a_1, \cdots, a_k)$ $\boxed{a_i > 0}$
- **length** k : number of « generating » simplices
- **dimension** : $\sum_{i=1}^{k} a_i$

Particular simploids:

- **Simplices** length = 1
- **Cubes** length = dimension

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Semi-Simploidal Sets

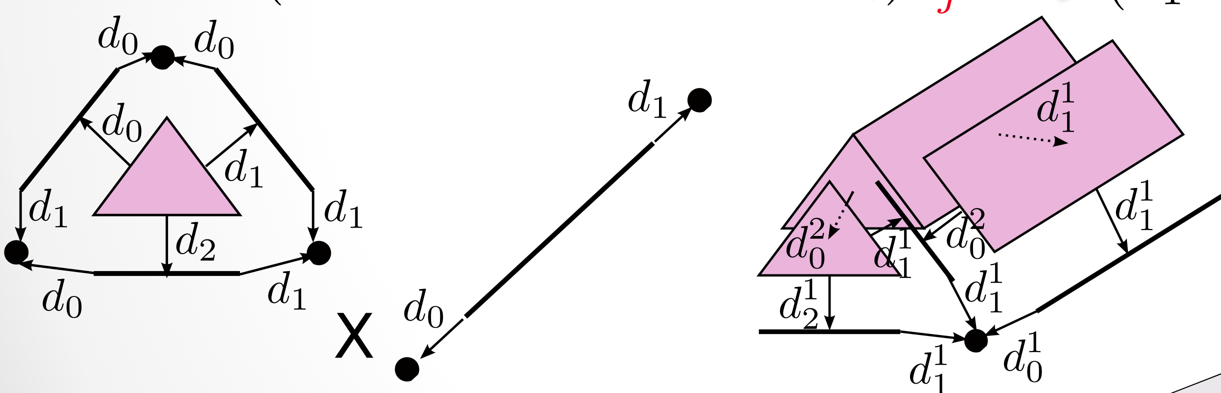
Semi-Simploidal Sets [Peltier et al. 09]

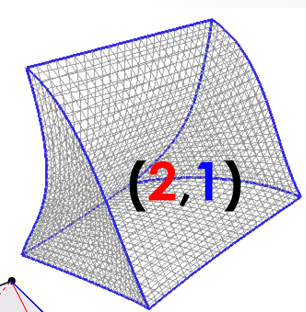
- set of **abstract simploids** $K = \{K^i\}_{i \in [0..n]}$
- **type operator** $\mathcal{T} : K \mapsto \bigcup_{i=0}^{\infty} \mathbb{N}^{*i}$

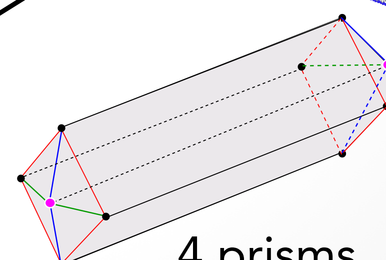
$$\sigma\mathcal{T} = (a_1, \ldots, a_n)$$

- **face operators** $d_j^i : K^i \to K^{i-1}$ satisfying **constraints**

$$(\sigma_1 \times \cdots \times \sigma_i \times \cdots \times \sigma_n)d_j^i \longrightarrow (\sigma_1 \times \cdots \times \sigma_i d_j \times \cdots \times \sigma_n)$$







➢ **Insures topology consistency**

Geometric Modeling with Bézier spaces

- Handling Subdivided objects
  - Combinatorial structure **(topology)** **+ operations**
  - Bézier Embedding **(geometry)**

Outline

- ✓ Context & Goal
- ✓ Filling in the quad mesh
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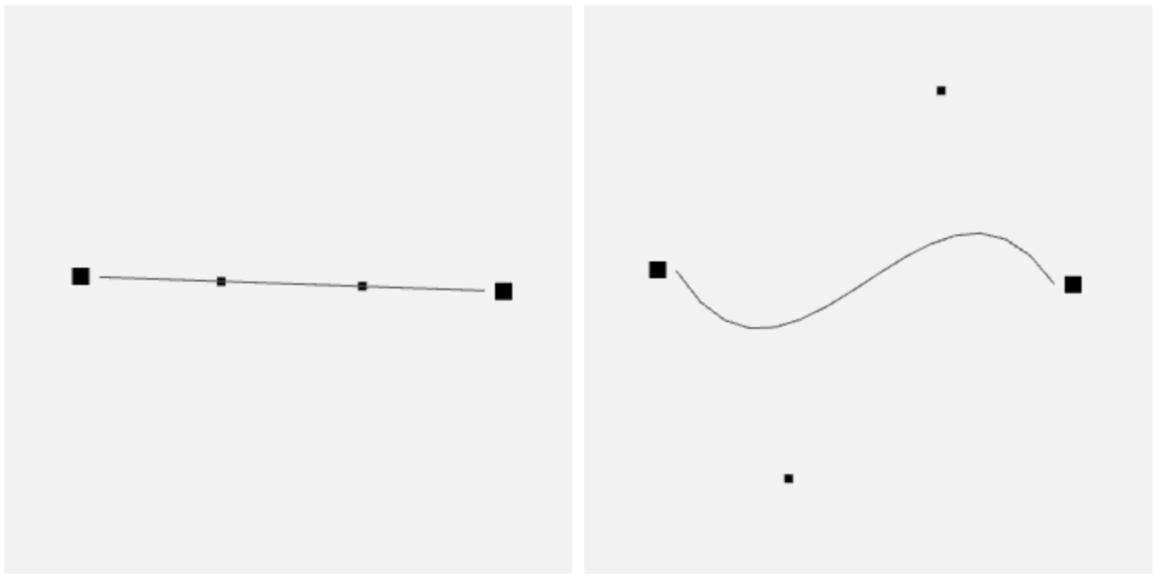
Non linear : Bézier

- A 1D-object is embedded into 3D as a Bézier curve

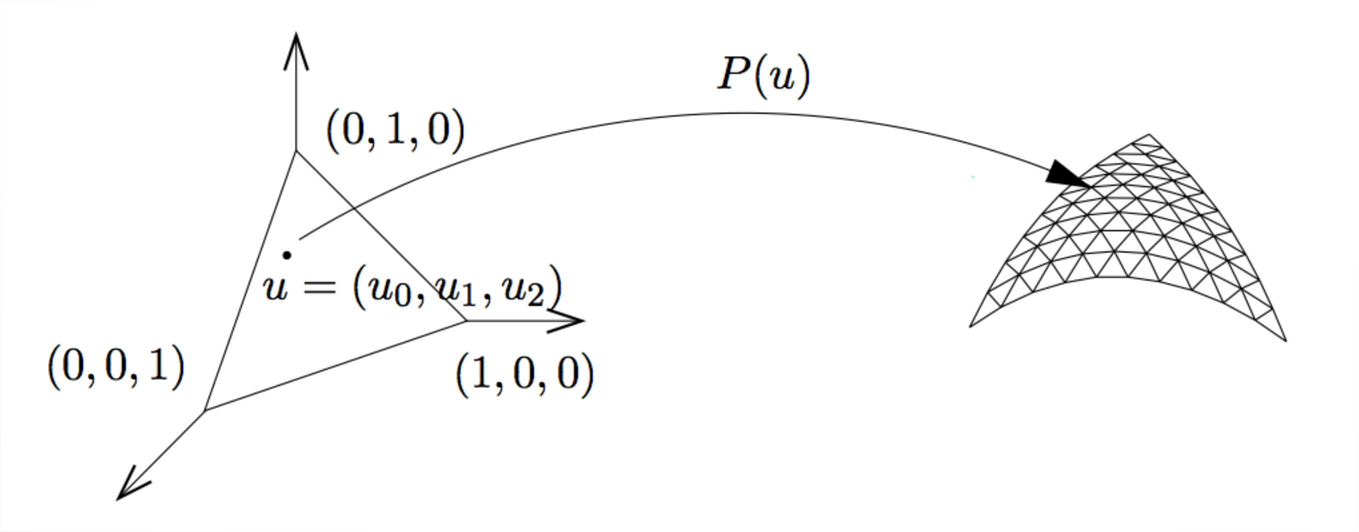


Simplicial Bézier Spaces

- Bézier simplex [Farin 2002] of dimension i and degree d

$$P(u) = \sum_{\alpha \in \Gamma_d^i} P_\alpha B_\alpha^d(u)$$

  - P_α Control points
  - multi-indices $\Gamma_d^i = \{\alpha = (\alpha_0, \cdots, \alpha_i) \mid \alpha_0 + \cdots + \alpha_i = d\}$
  - Multivariate Bernstein Polynomials $B_\alpha^d(u) = (\frac{d!}{\alpha_0! \cdots \alpha_i!}) u_0^{\alpha_0} \cdots u_i^{\alpha_i}$



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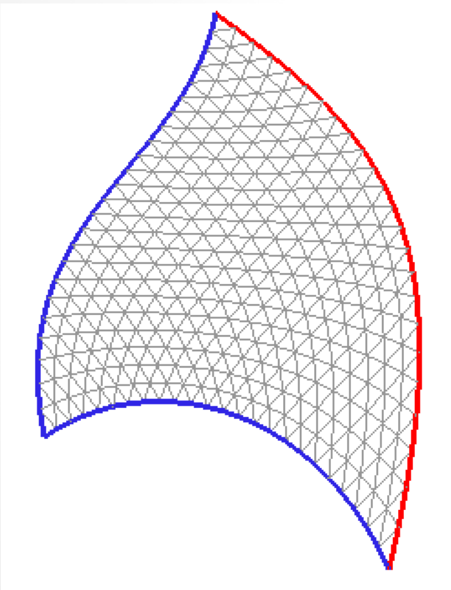
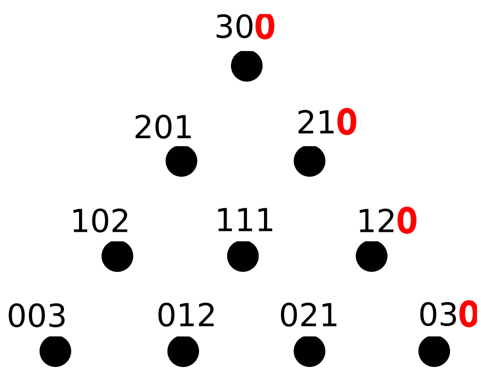


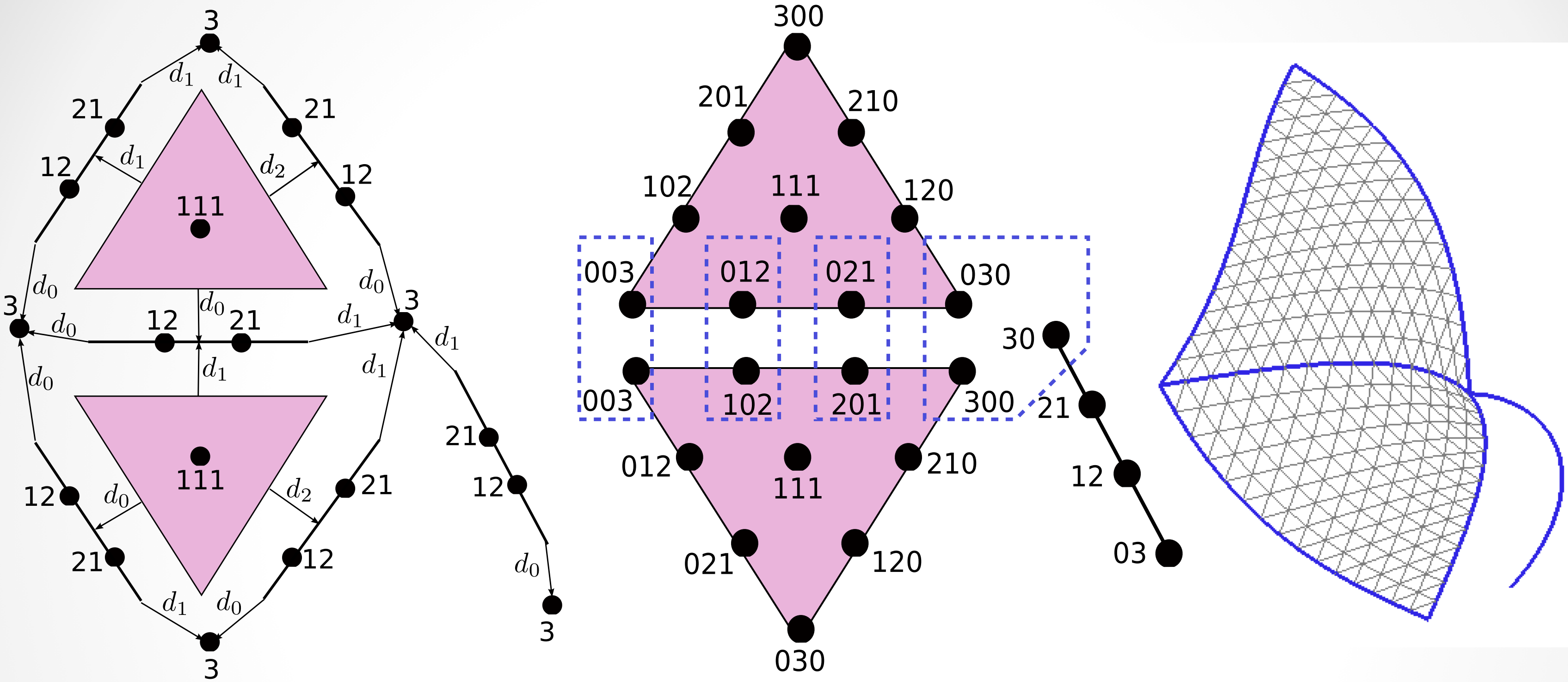


Dimension 2, degree 3

- Property of multi-indices:
  - **Control points** $P_{\alpha_0 \cdots 0 \cdots \alpha_i}$

  define a Bézier simplex of dimension i-1

Simplicial Bézier Spaces



Semi-simplicial set of dimension n

$K = \{K^i\}_{i \in [0..n]}$ set of **abstract simplices**

$d_j : K^i \to K^{i-1}$ **face operators**

$d_j d_l = d_l d_{j-1}, j > l$ **commutation properties**

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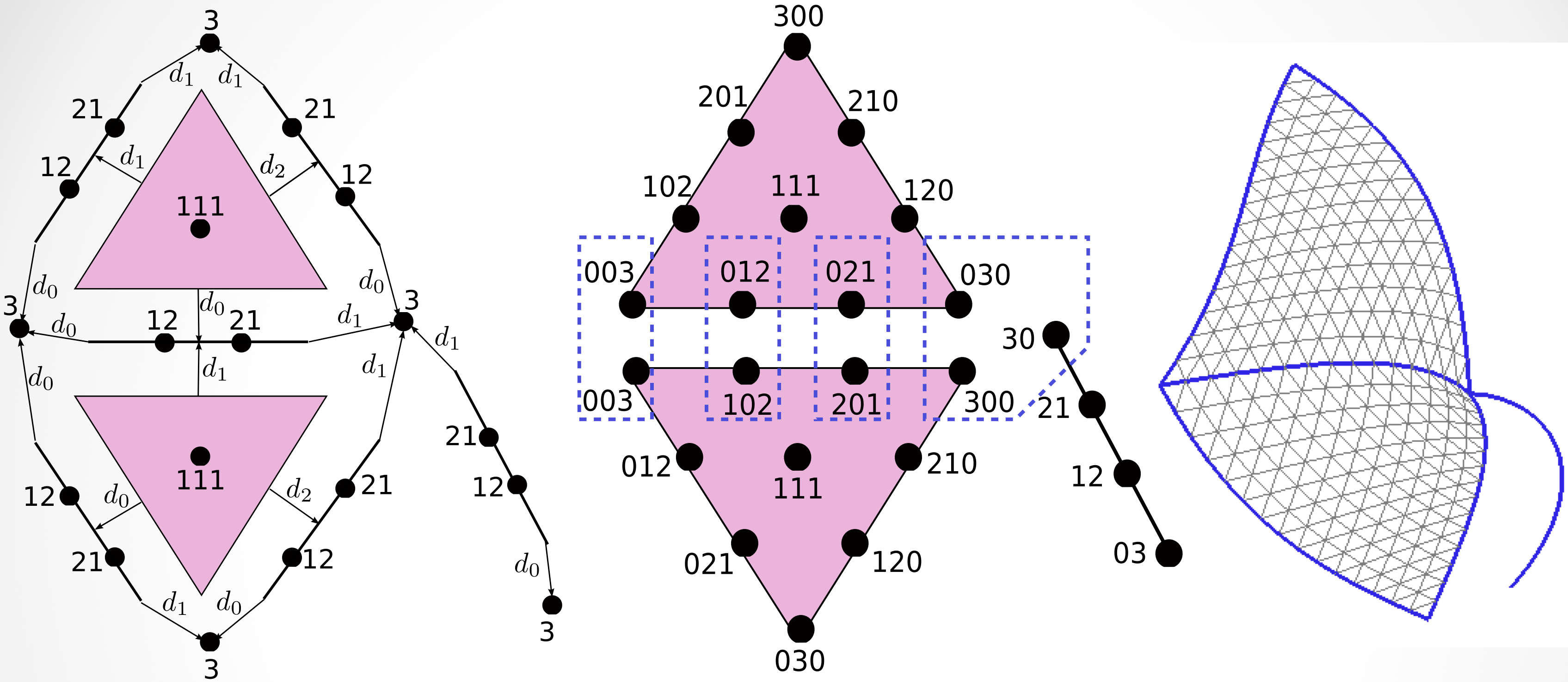
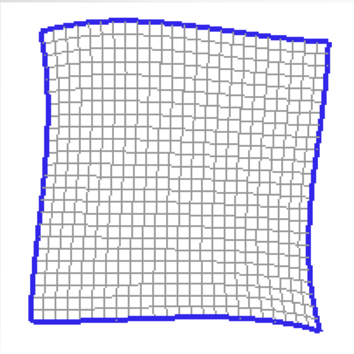
Simplicial Bézier Spaces

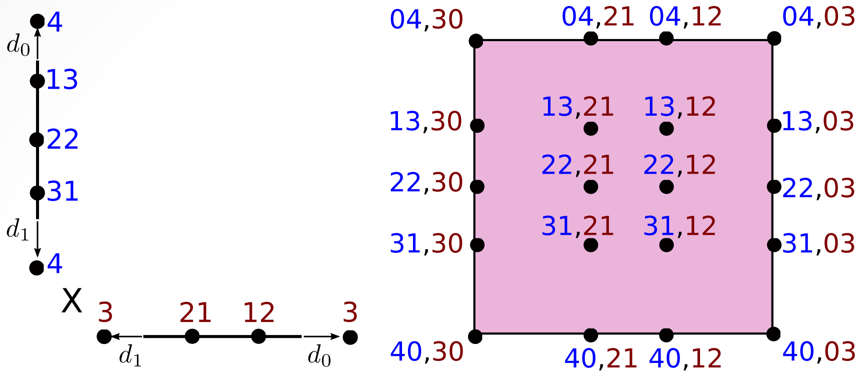


Each simplex stores its « proper » control points

- **No redundancy**
- **Keeps topology consistency**
- **Multi-indices can be retrieved using face operators**

Cubical Bézier Spaces





Semi-Cubical sets [Brown Higgins 81]

- abstract cubes
- face operators

$$(e_1 \times e_2) d_j^1 = (e_1 d_j \times e_2)$$
$$(e_1 \times e_2) d_j^2 = (e_1 \times e_2 d_j)$$

- commutation properties

Simploidal Bézier Spaces

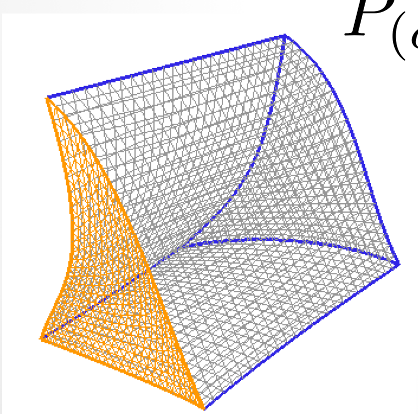
Bézier Simploid [DeRose et al. 93]

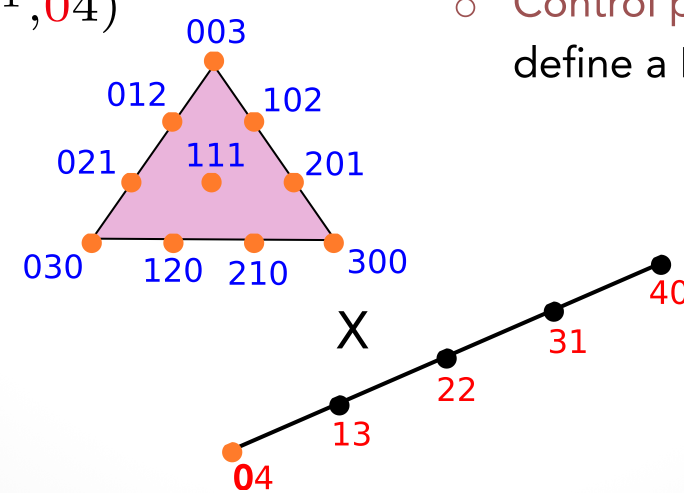
of type $(a_1, \ldots, a_n)$ and degree $(d_1, \ldots, d_n)$:

$$P(u^1, \ldots, u^n) = \sum_{\alpha^1 \in \Gamma^{a_1}_{d_1}} \cdots \sum_{\alpha^n \in \Gamma^{a_n}_{d_n}} P_{(\alpha^1, \ldots, \alpha^n)} B^{d_1}_{\alpha^1}(u^1) \times \ldots \times B^{d_n}_{\alpha^n}(u^n)$$

control points $\{P_{(\alpha^1, \cdots, \alpha^n)}\}$ identified by **tuples of multi-indices**

$P_{(\alpha^1, 04)}$

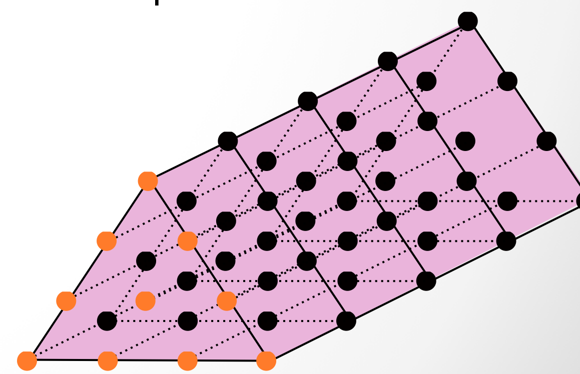




Property of tuples of multi-indices:

- Control points

  define a Bézier simplex of dimension i-1



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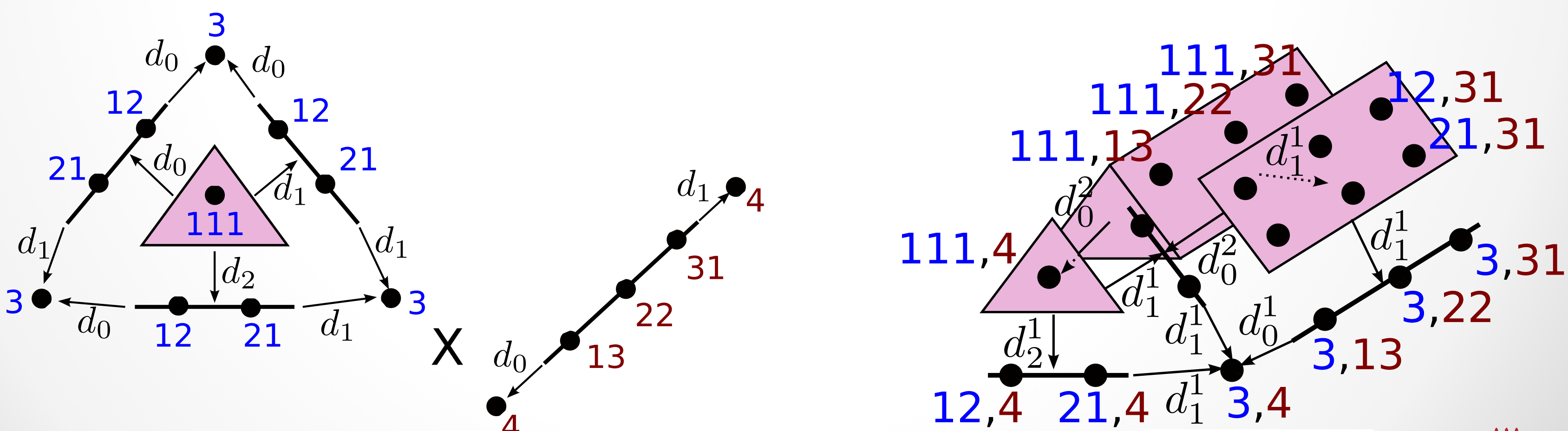
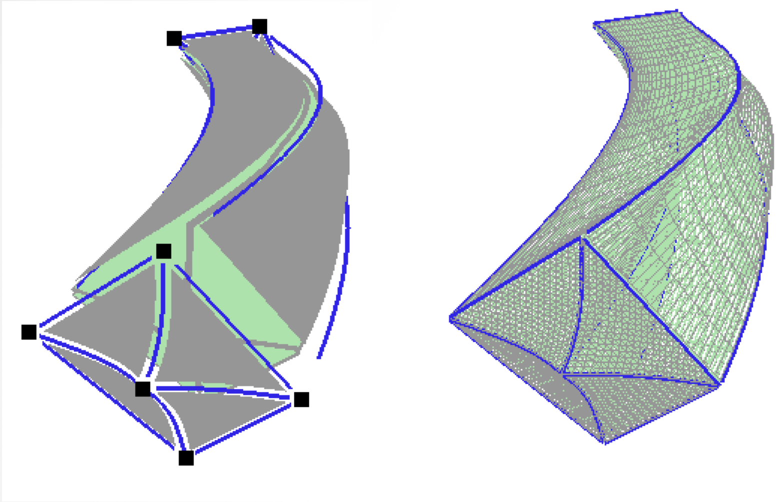


Branches & Kites

Branch : Semi-simploidal Set
(assembly of 4 prisms)



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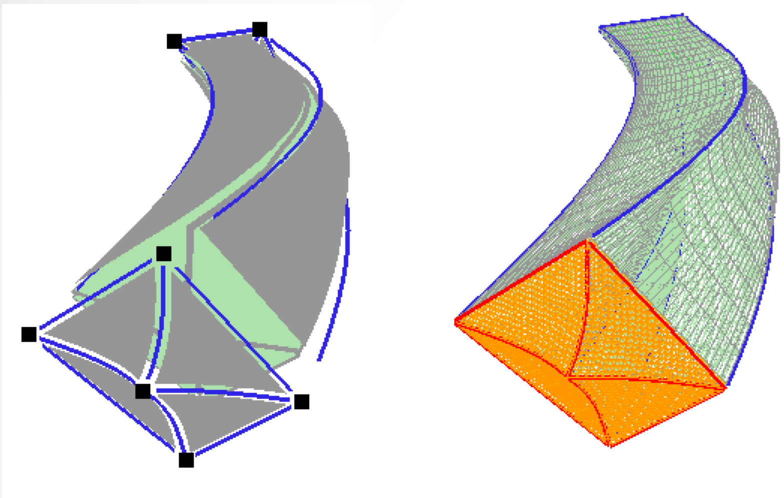
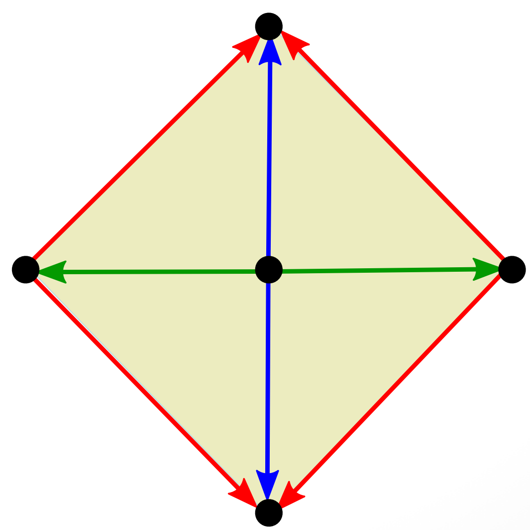
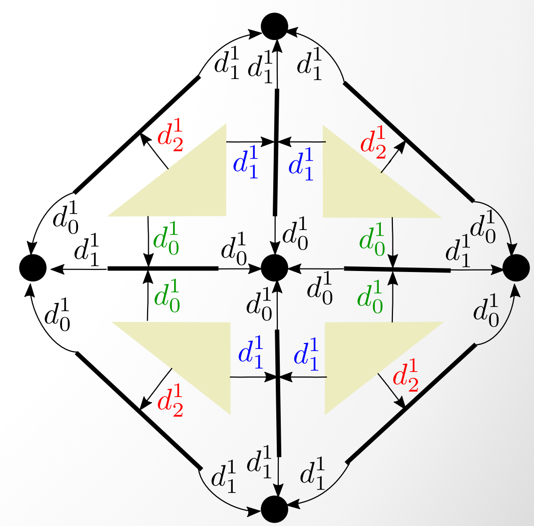
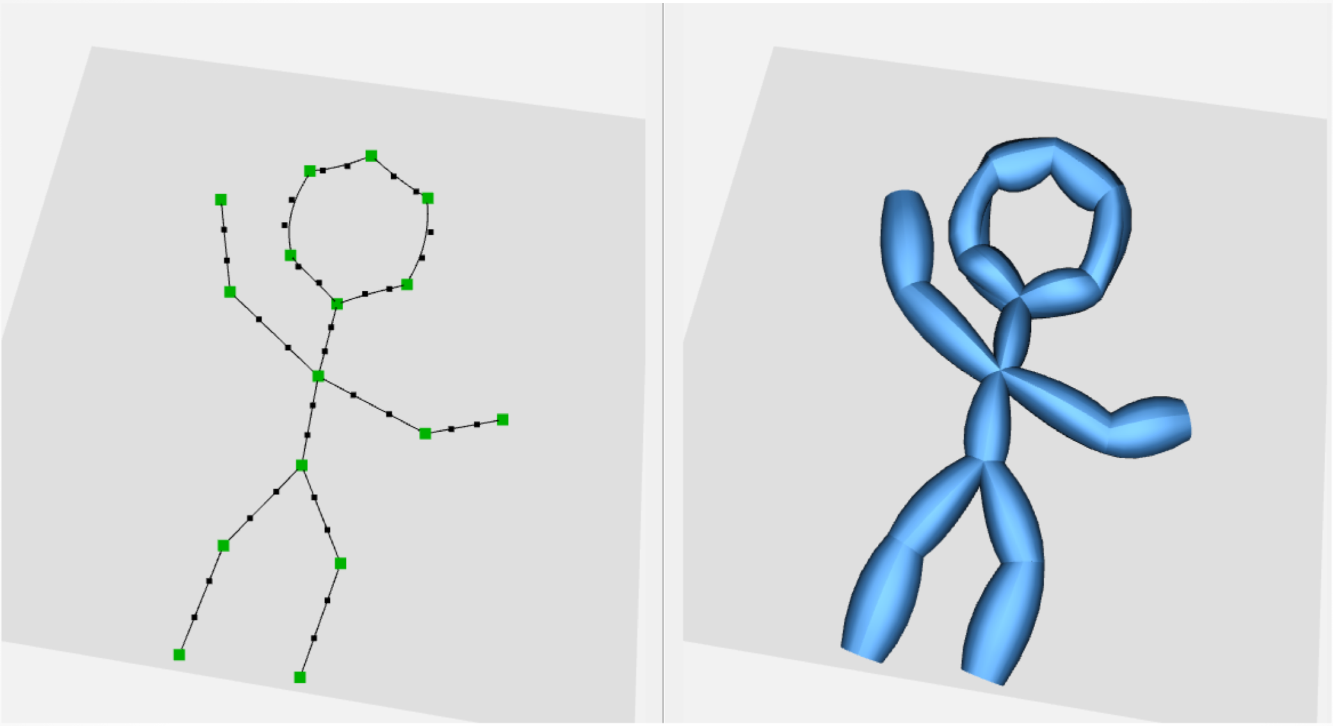


Kite : Branch end
(assembly of 4 triangle)





Generating a free from volume



- Each branch is a 4-prism (semi-simploidal Bézier set)
- Each prism is a Bézier volume (simploid) of degree 3
- **Topology consistency**, C^0 at the joints

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- ✓ Context & Goal
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- ✓ Handling non linear, free form 3D objects
- Joining the branches in 3D (on going work)
- Conclusion – Future Work

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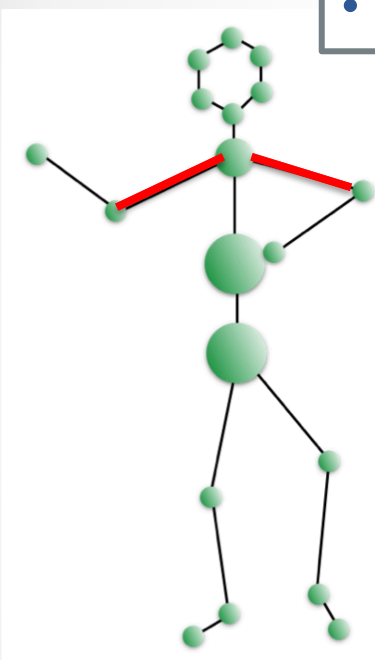
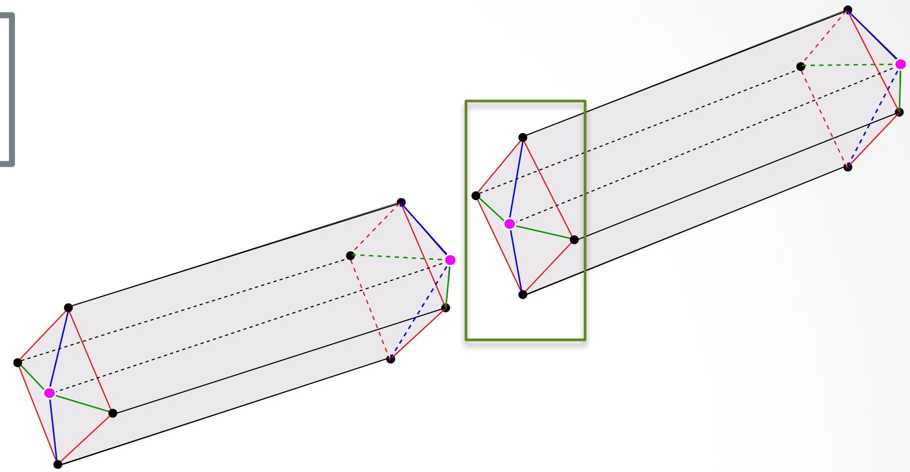
Joining the branches

Adding a branch

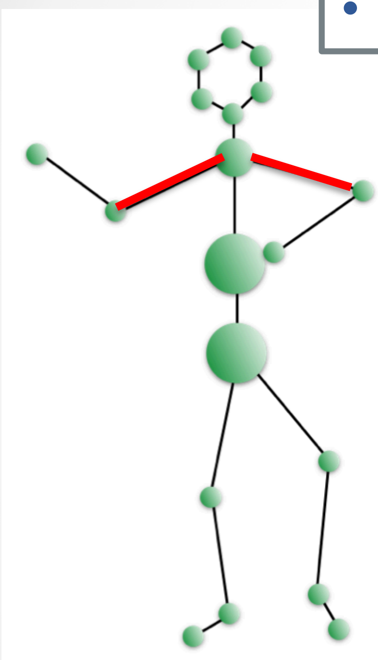
- free kite

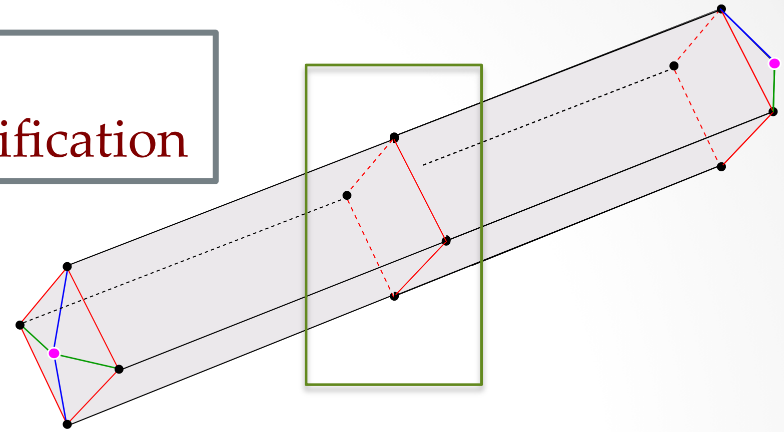




Joining the branches

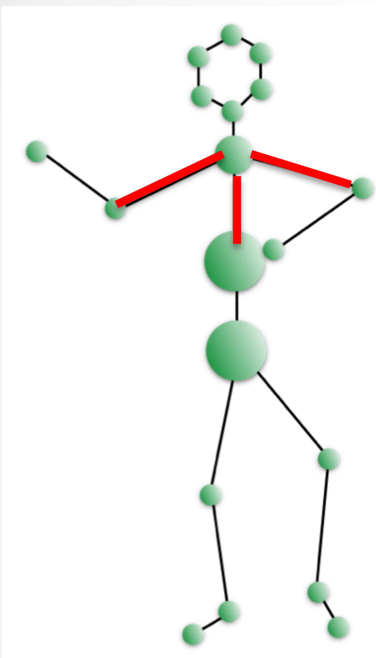
Adding a branch
- free kite : kite identification

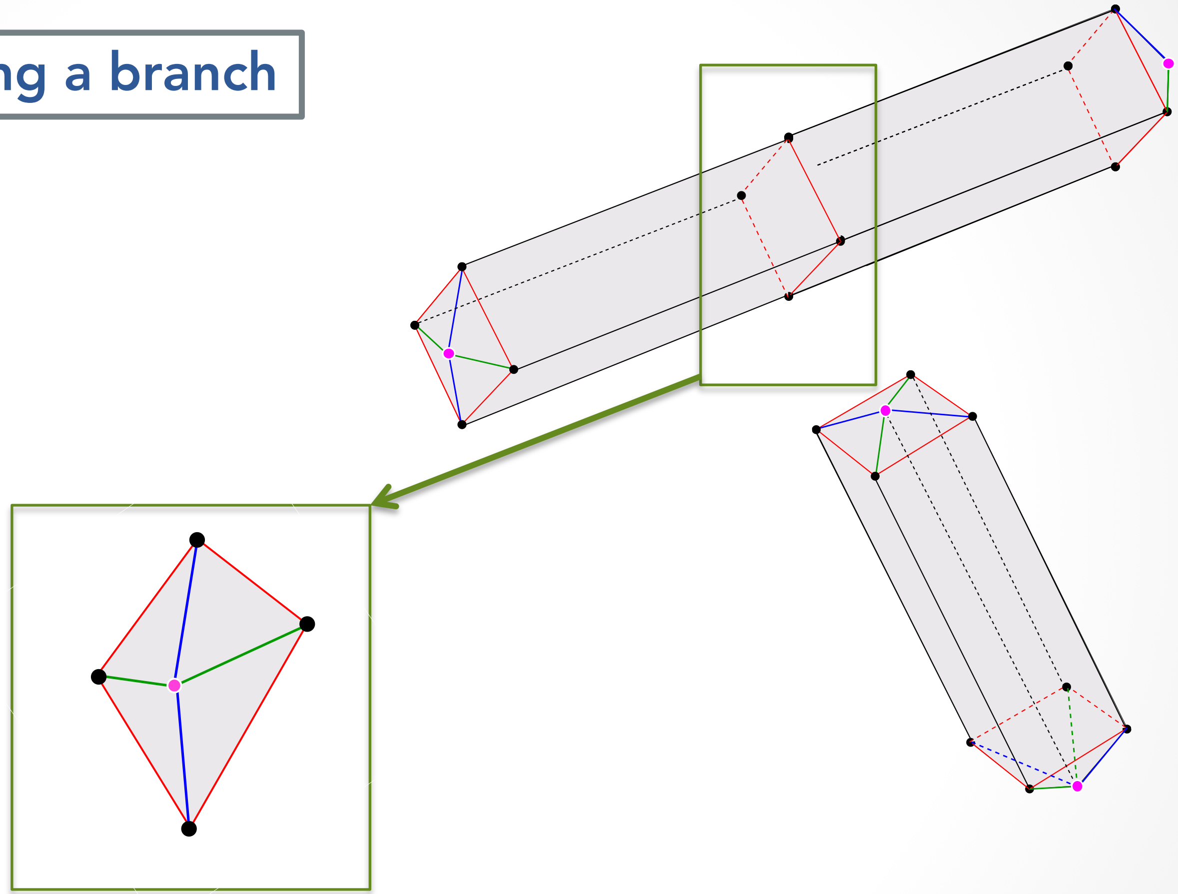




Joining the branches

Adding a branch

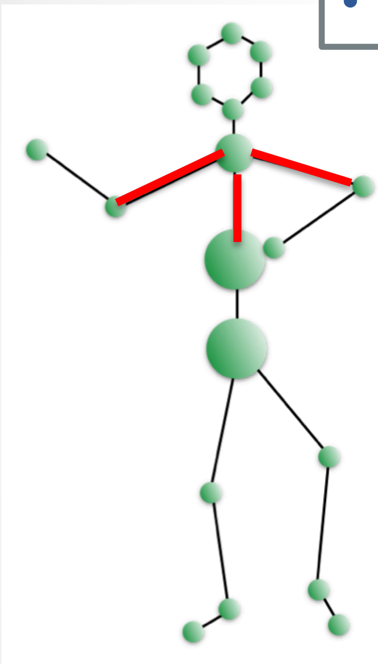


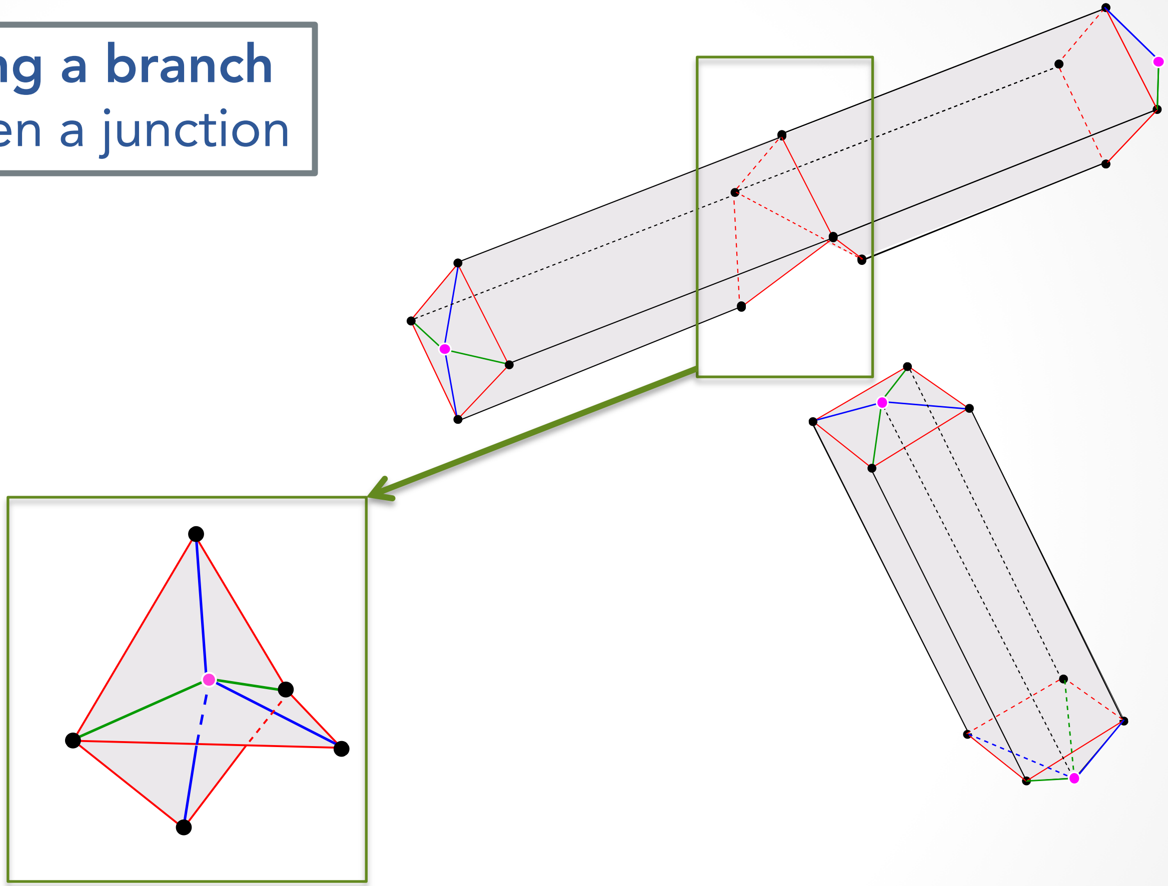


arity 2

Joining the branches

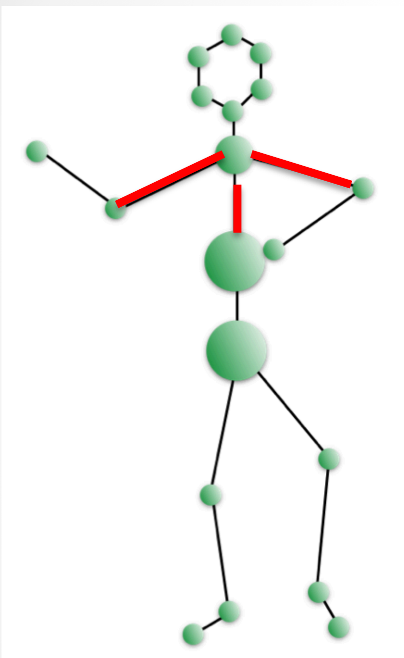
Adding a branch
- open a junction



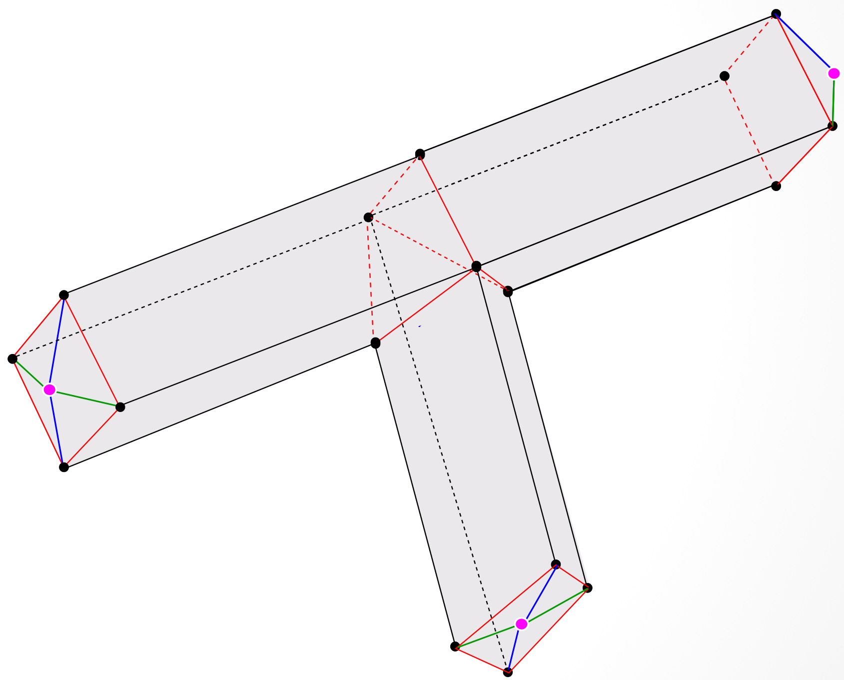


arity 3

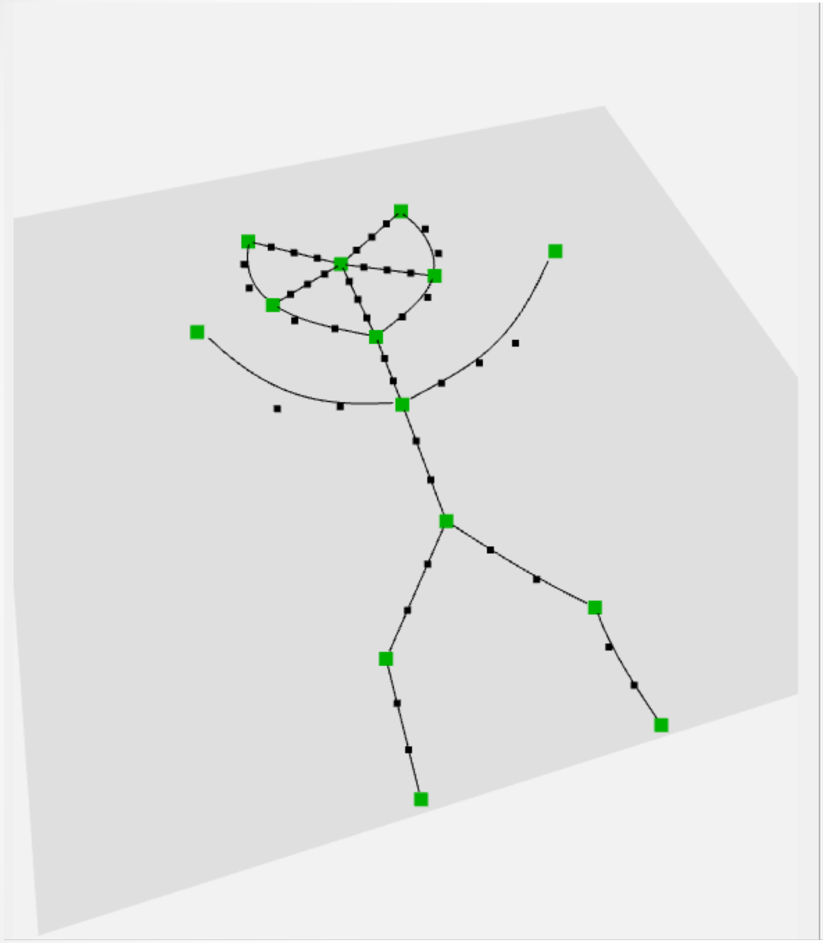
Joining the branches



Volume Mesh : assembly of branches (identification of kytes) **built incrementaly**

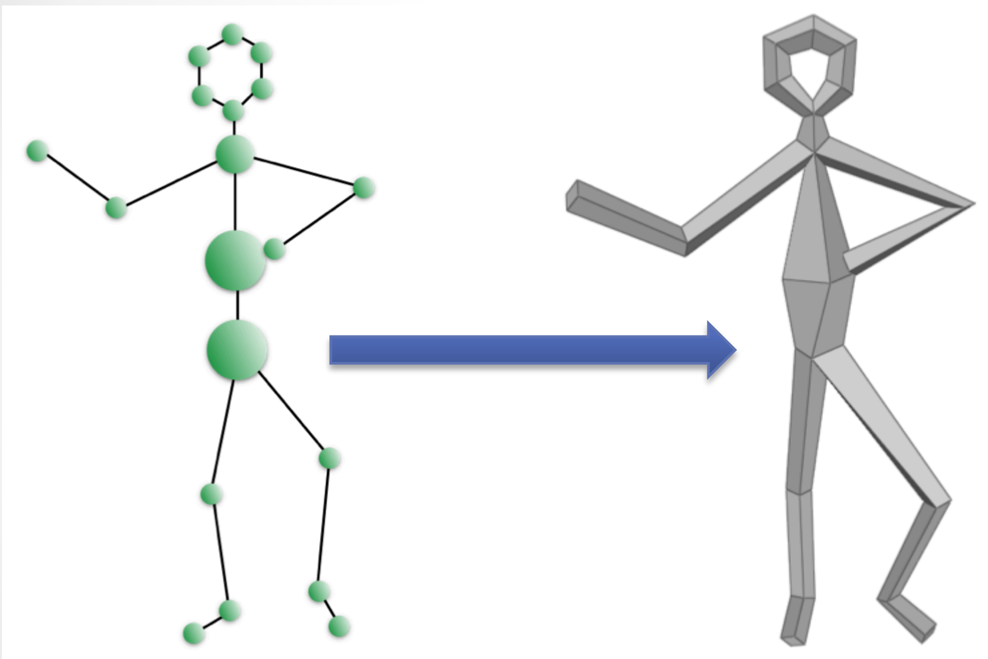


Joining the branches in 2D



Joining the branches in 3D

Scaffolding a Skeleton, Panotopoulou et al., Research in Shape Analysis, 2018



Skeleton **Quad mesh**

4 quads around each edge

-> proves minimal for regular branches

-> constructs a quad mesh around the joint in 3D (more general than 2D)

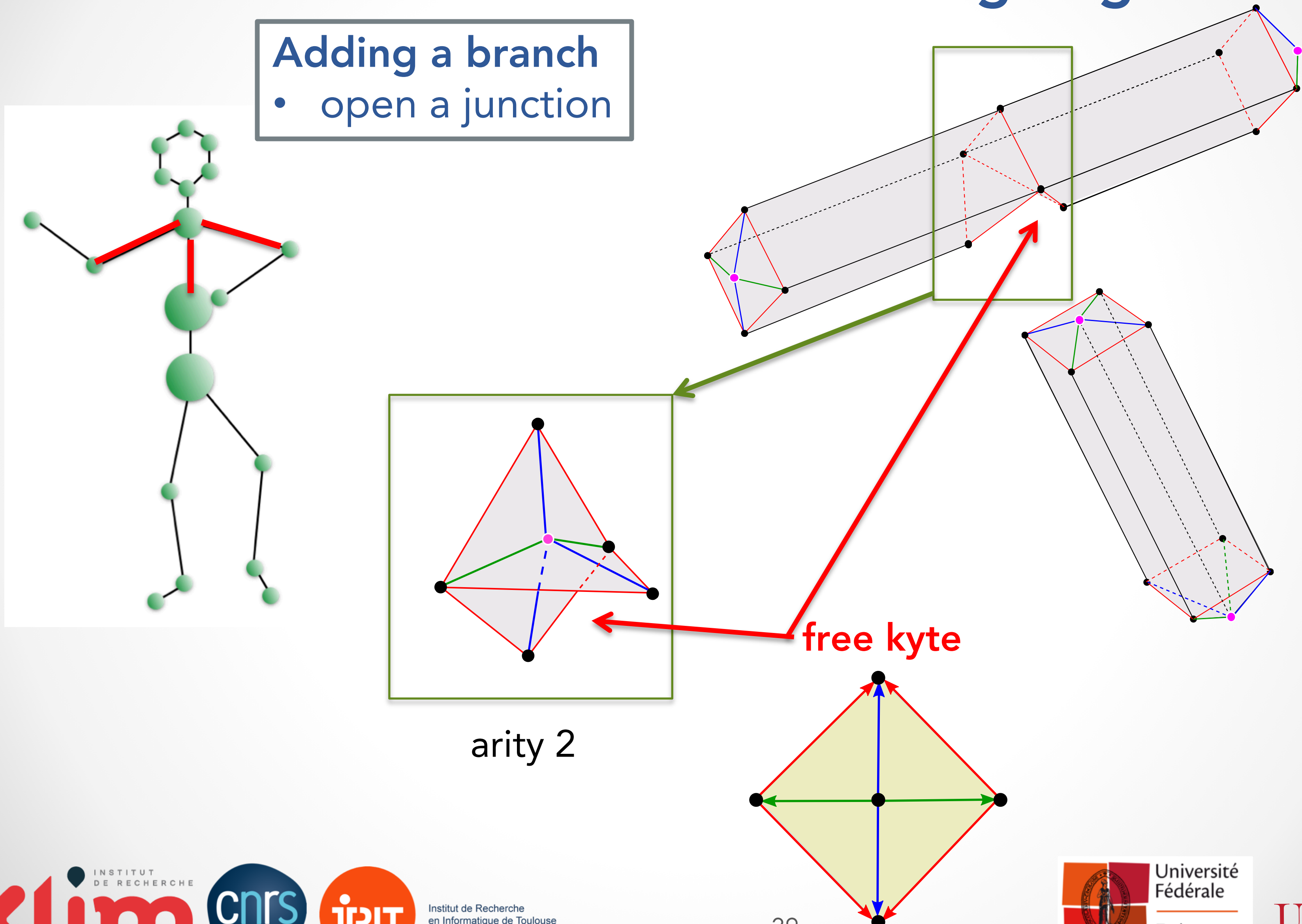
BUT the proposed construction does not generate all possible configurations

Joining the branches in 3D

On going work

Adding a branch

- open a junction

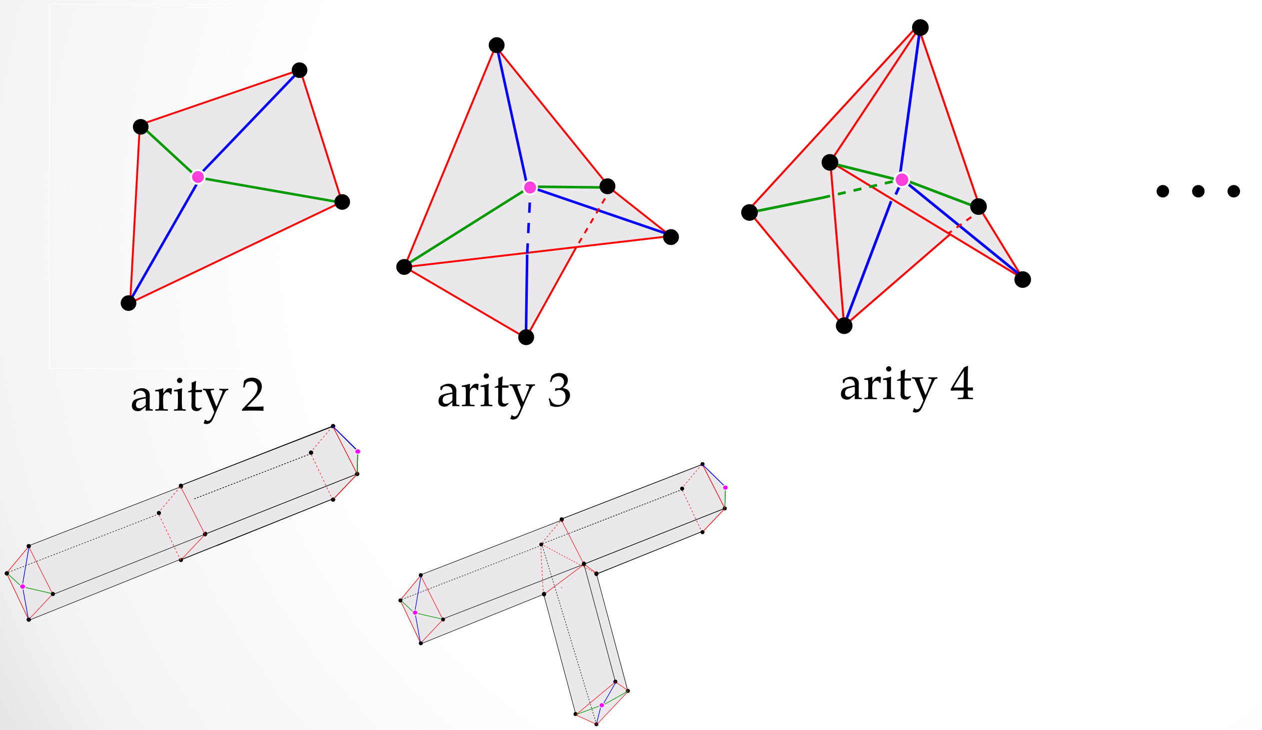


Joining the branches in 3D

On going work

Generalizes to arbitrary number of branches in 3D

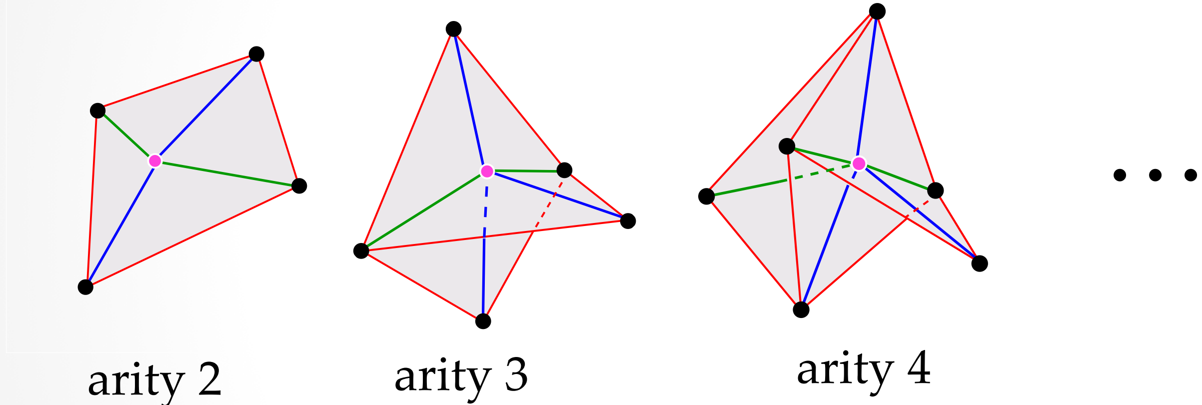
➢ **Any arity can be handled (+ cells orientation)**

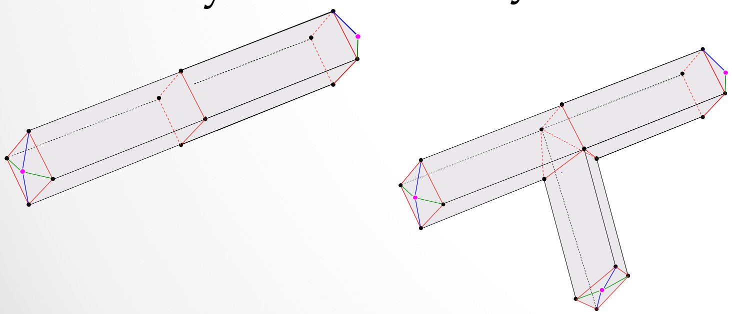


Joining the branches in 3D

On going work

We show that all (non degenerate) topological joining configurations may be generated through an iterative kite opening process





Joining the branches in 3D

We showed that :

- Incremental kite opening leads to any configuration !

Future work (current work of Damien !)

- Geometric embeding for the points
  - Linear setting
  - Non linear setting
- Independance to branch order
- Convexity of the branches

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Conclusion

Conclusion :

We propose an algorithm for Bézier volume mesh generation from skeleton

- 4 quads around each edge
- only one volume cell type (prisms)
- border is a Bézier **surface mesh**
- **Topology** : no cracks !

Special thanks to the *Poitevins*

Benoît Gougeon, Clément Castin, Damien Aholou et Valentin Fredon

For the software development of the volume model (Master project)

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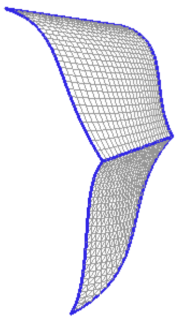
Future work - long term

Model smoothness

- C^0 is « for free »
- Splines,…



Animation / Simulation

- Motion on control points
- Continuous motion over the mesh
- Phycical constraints

Thank you

- Question ?

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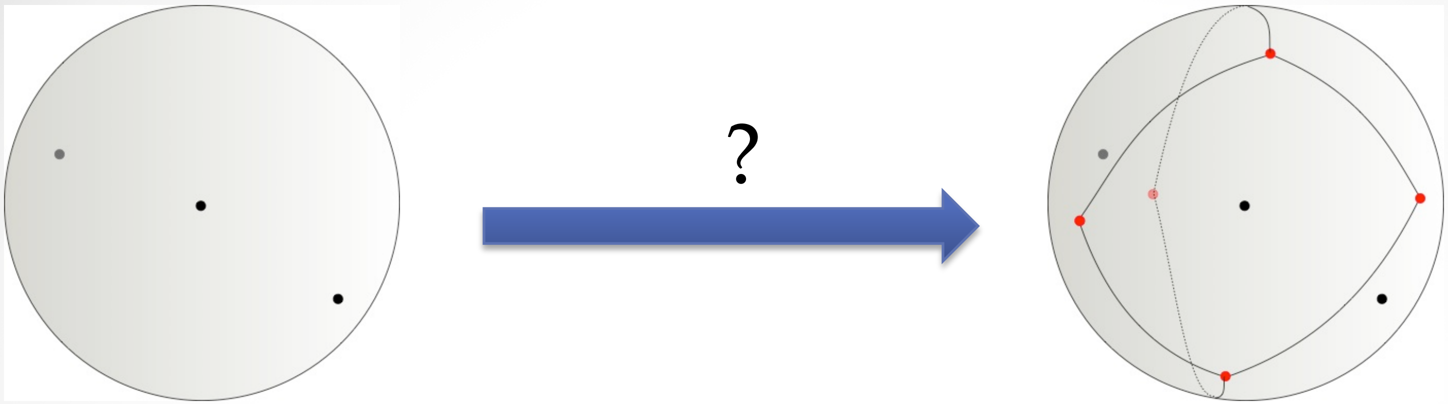
Question



Given a sphere with k points

Canonical Quad mesh with k quads

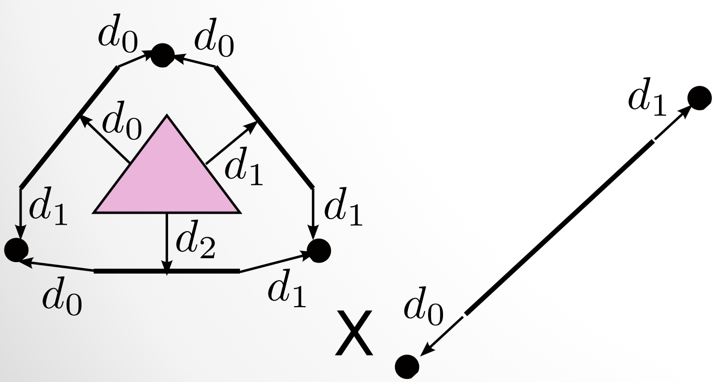
In this case, we can obtain a canonical Volume Mesh

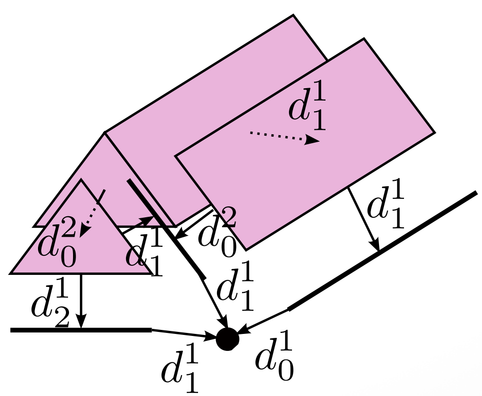
Semi-Simploidal Sets

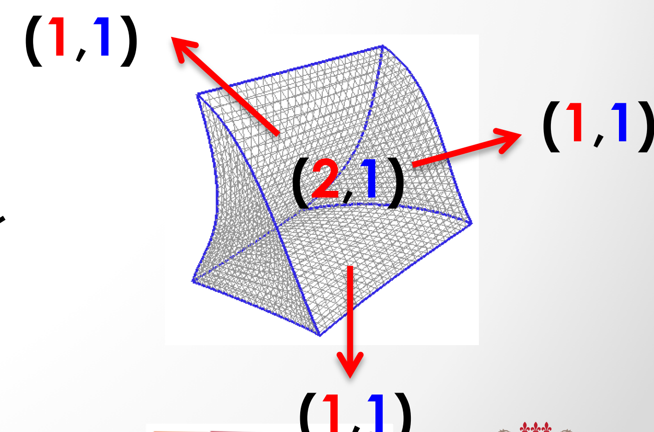
Semi-Simploidal Sets [Peltier et al. 09]

- set of **abstract simploids** $K = \{K^i\}_{i \in [0..n]}$
- **type operator** $\mathcal{T} : K \mapsto \bigcup_{i=0}^{\infty} \mathbb{N}^{*i}$
  $$\sigma\mathcal{T} = (a_1, \ldots, a_n)$$
- **face operators** $d_j^i : K^i \to K^{i-1}$ satisfying **constraints**

$$(\sigma_1 \times \cdots \times \sigma_i \times \cdots \times \sigma_n) d_j^i \longrightarrow (\sigma_1 \times \cdots \times \sigma_i d_j \times \cdots \times \sigma_n)$$







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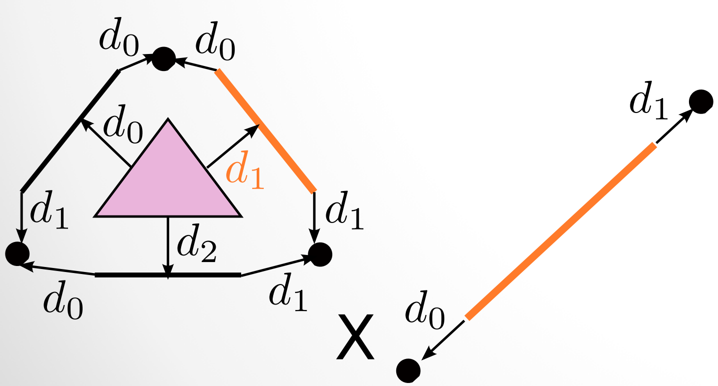

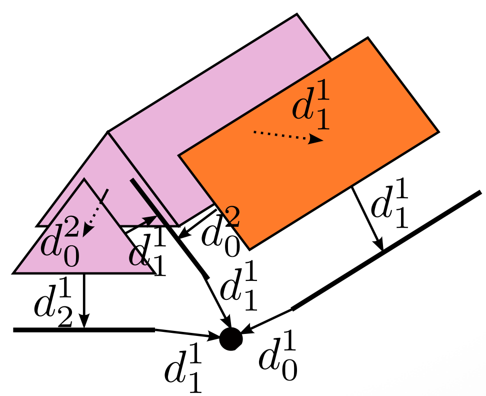

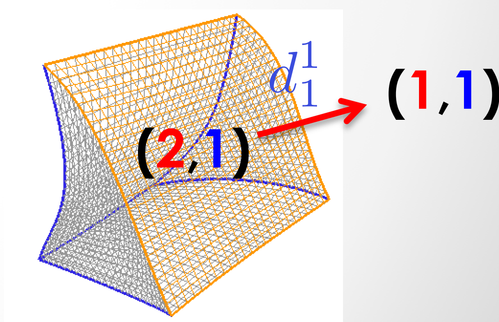

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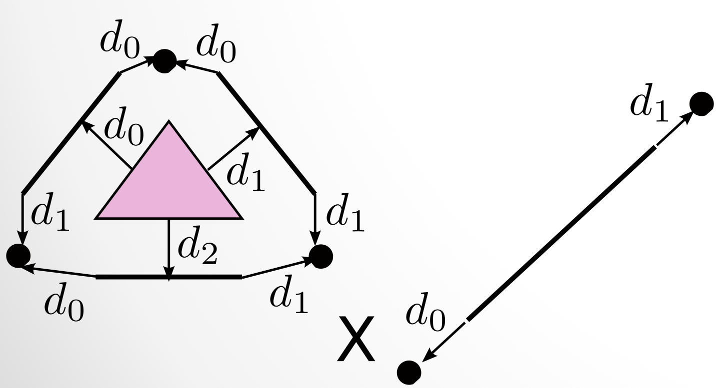

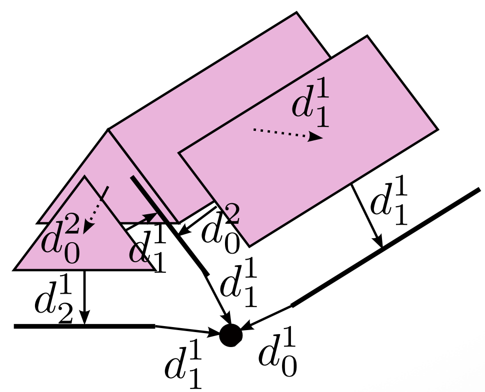

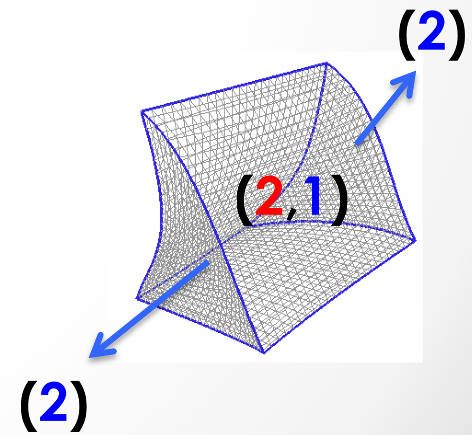

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