Boundary-based Mesh Partitioning for Geometrical Product Specifications and Verification

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GTMG : Journées
du Groupe de Travail
en Modélisation Géométrique

LORIA Nancy

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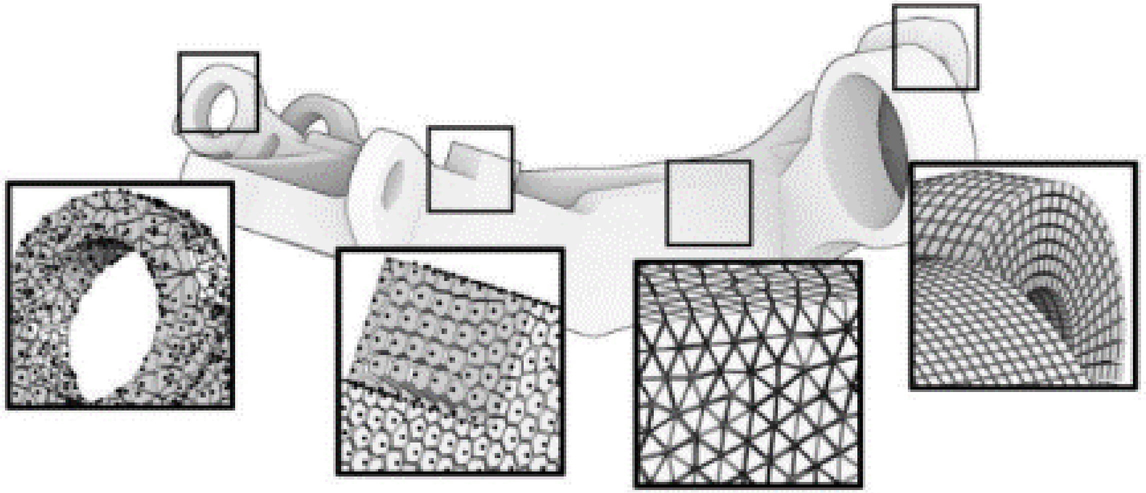
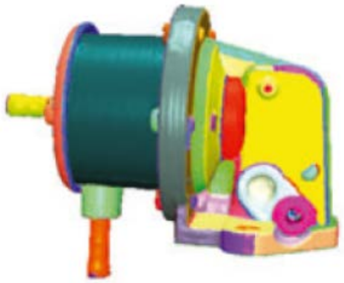


Content

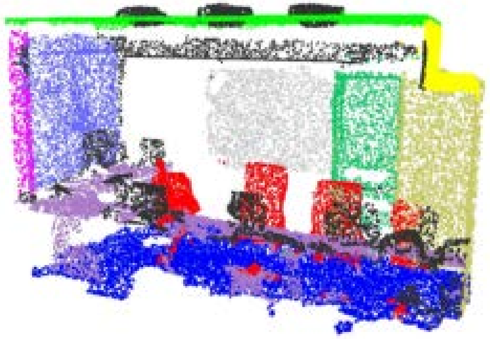
Introduction

- Partition applications in
  - Reverse engineering
  - Civil engineering
  - Computer vision
  - …

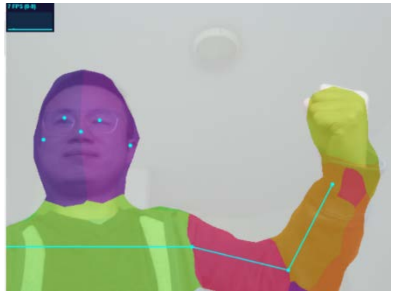


Geometry design [SWK07]

Building reconstruction [SWK07]



Semantic segmentation [QSM17]



body recognition [Google BodyPix]

Introduction

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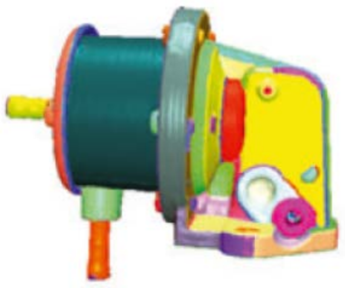
  - Reverse engineering
  - Civil engineering
  - Computer vision
  - …



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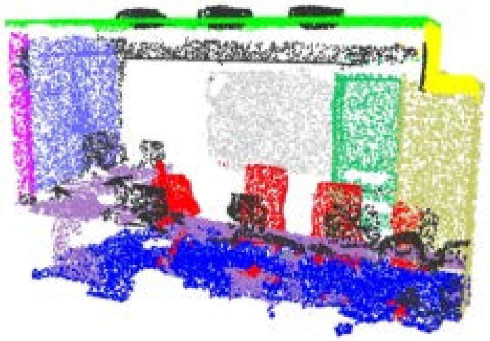

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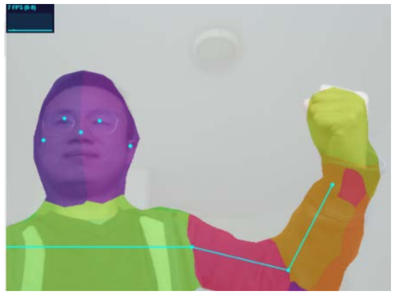
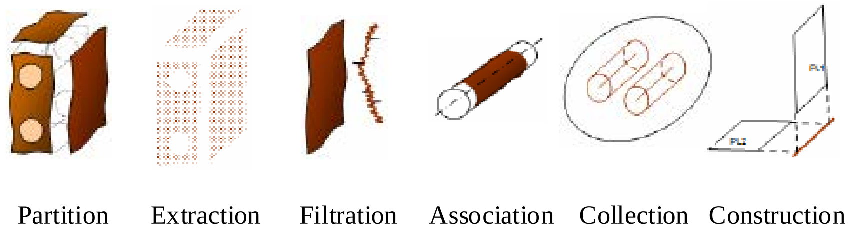


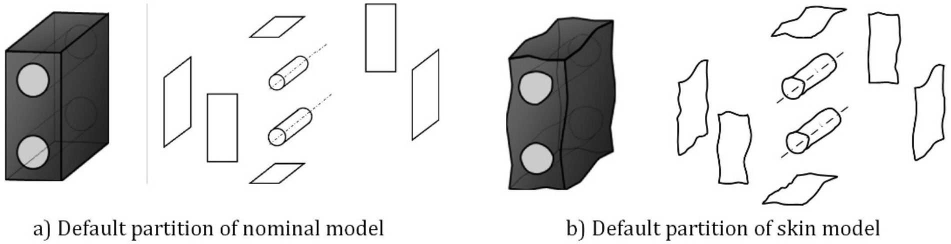
body recognition [Google BodyPix]

- **Partition for Geometrical Product Specifications and verification(ISO GPS)**
  - “Feature operation used to identify a portion of a geometrical feature belonging to the real surface of the workpiece or to a surface model of the workpiece” [ISO 17450-1:2011]

Introduction

- Geometrical Product Specifications and verification (ISO GPS)
  - Define both tolerancing (for specification) and metrology (for verification) practices
- Partition
  - One of the feature operations to obtain ideal/non-ideal features
  - Specify the geometry of a product at meso level
  - Decompose the object into independent surface portions
  - Ongoing standardization efforts within ISO GPS
    - 18183-1, Partitioning – Part 1: Basic concepts
    - 18183-2, Partitioning – Part 2: Nominal model
    - **18183-3, Partitioning – Part 3: Methods used for Specification and Verification**





ISO/WD 18183-3:2018(E)

ISO TC 213/SC/AG 12

Secretariat: XXXX

Geometrical product specifications (GPS) — Partitioning — Part 3: Methods used for Specification and Verification

Research Objective and Questions

- Mesh partitioning method for ISO GPS

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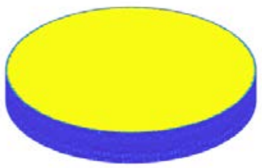
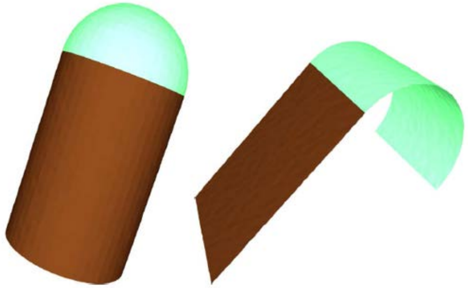


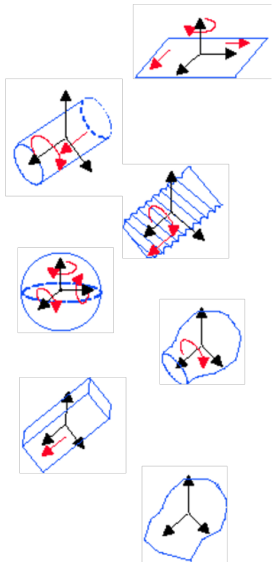
Sharp edges



Non-sharp edges

- Find the natural boundaries
  - Edges where an abrupt change of point deferential geometry properties occurs
  - The boundaries among smoothly-connecting regions
- Address invariance classes

	ISO GPS Invariance Classes (Lower-order Kinematic Pairs)	Kinematic Degrees of Freedom
1	Planar	2 Translations and 1 Rotation
2	Cylindrical	1 Translation and 1 Rotation
3	Helical	1 Translation and 1 Rotation, linked by pitch
4	Spherical	3 Rotations
5	Revolute	1 Rotation
6	Prismatic	1 Translation
7	Complex	None



Related work

- **Classification of existing partition/segmentation methods**
  - Edge detection
  - Region growing
  - Attributes clustering
    - Iterative clustering
    - Hierarchical clustering
  - Shape fitting
  - Spectral analysis
  - Deep learning

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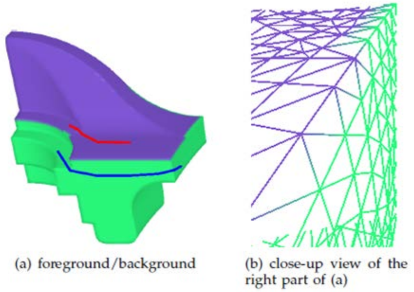

Edge detection method[ZZC10]

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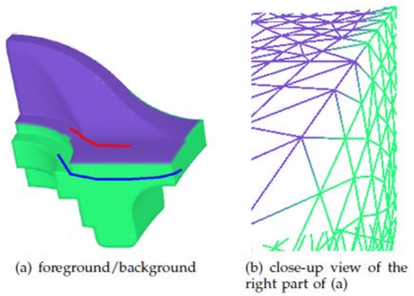

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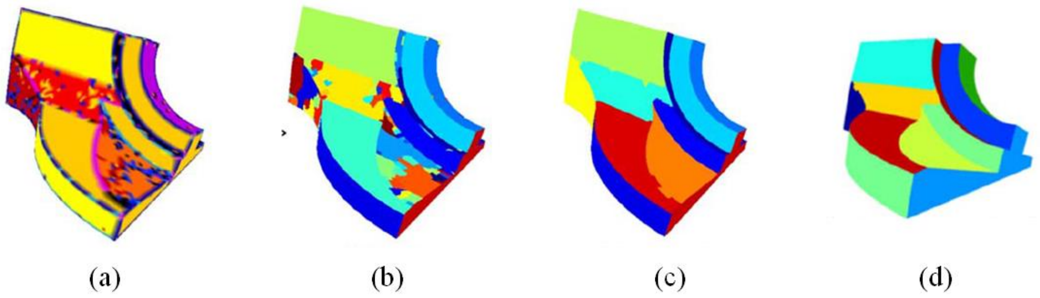

Region growing method [LBD05]

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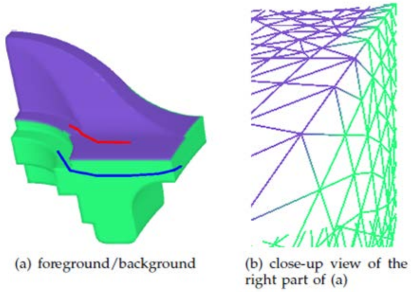

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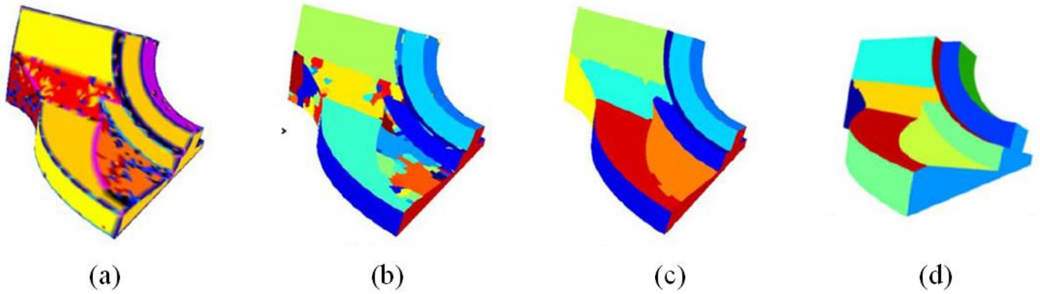

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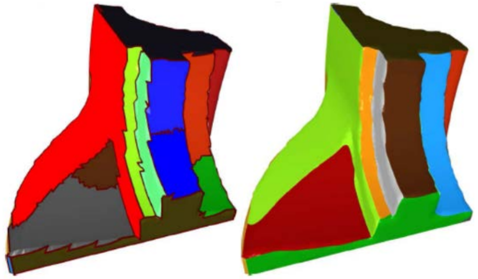

Attributes clustering [LHMR06]

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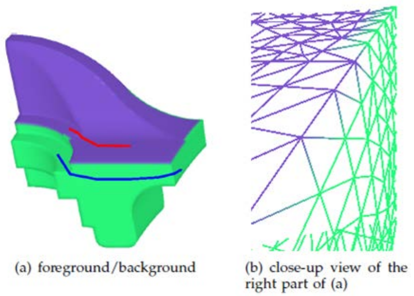

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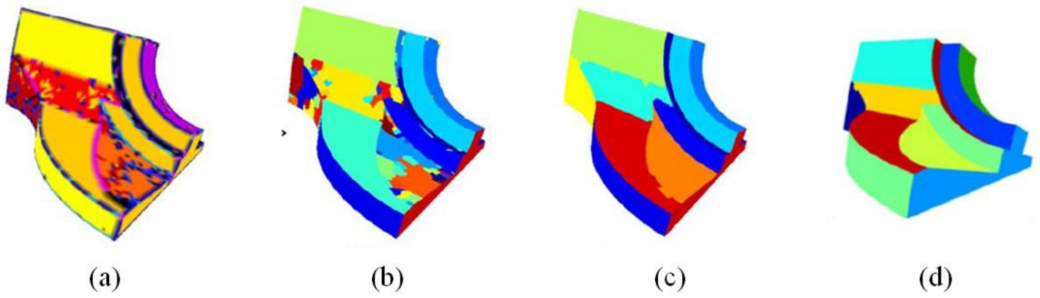

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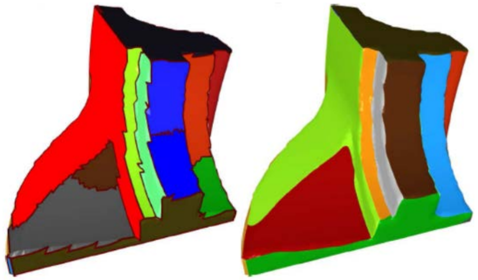

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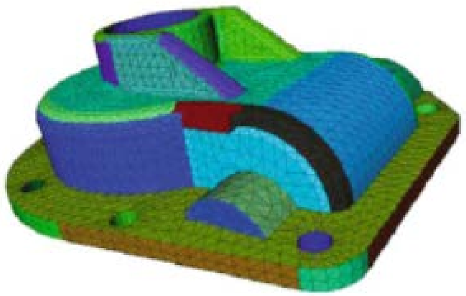

Shape fitting method[AFS06]

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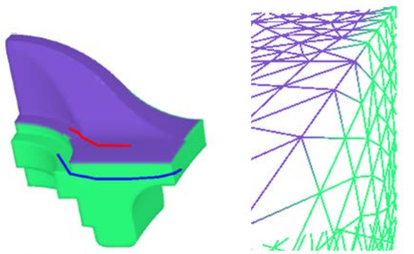
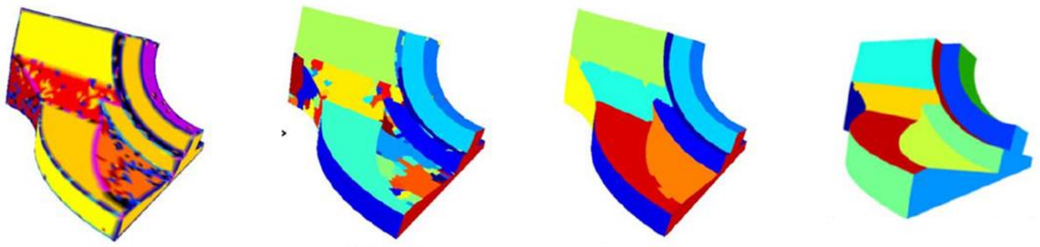


(a) foreground/background (b) close-up view of the right part of (a)

Edge detection method[ZZC10]



(a) (b) (c) (d)

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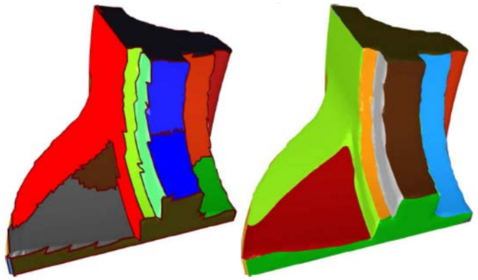

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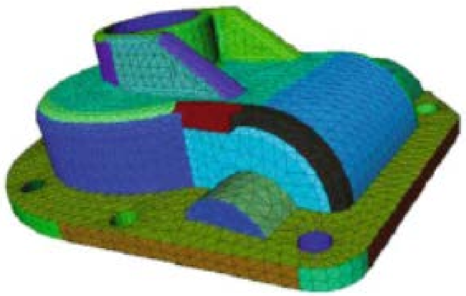

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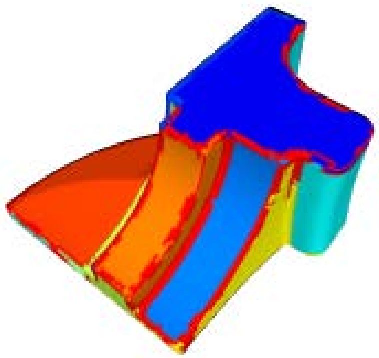

Spectral methods [WI18]

Related work

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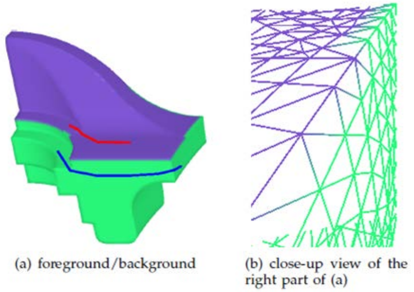

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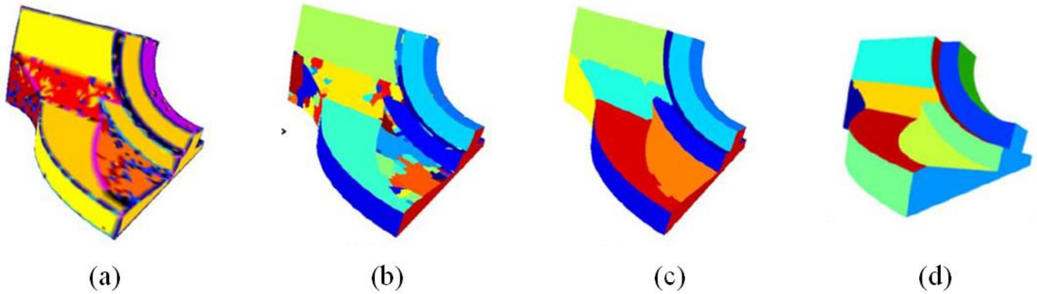

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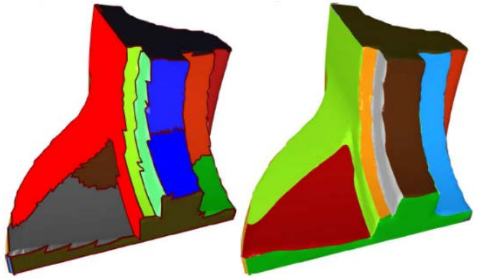

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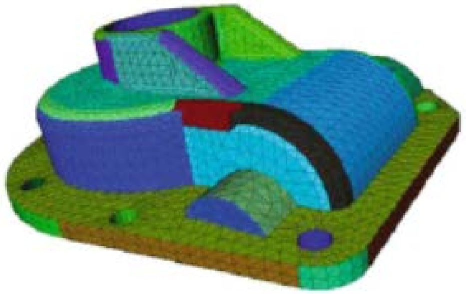

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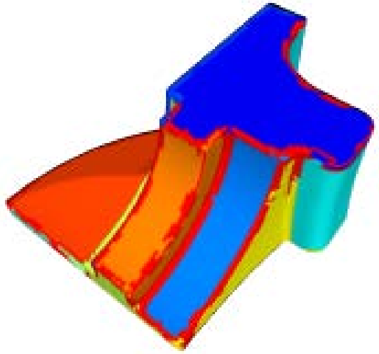


Spectral methods [WI18]

Deep learning method [QSM17]

Related work

Comparison of existing partition/segmentation methods

Methods	Characteristic	Specificity	Limitation
Edge Detection	▪ Locate the edges; ▪ Group the points inside	▪ High efficiency regarding sharp feature	▪ Sensitive to noise and density of point cl ouds; ▪ Unable to detect non-sharp edges
Region Growing	▪ Start with a seed point and grow while a pre-defined condition holds	▪ Commonly used due to simplicity ▪ Perform well regarding non-sharp features	▪ Partitioning results rely on the choice of t he initial seed points and the criterion to st op the process
Attributes Clustering	▪ Calculate the distance of each element to the specific region	▪ Perform well considering the CAD models	▪ Parameters (e.g. number of regions) are needed for iterative clustering; ▪ Post-processing is necessary for the hierarchical clustering
Shape Fitting	▪ Fit primitive shapes from the point cloud or mesh	▪ Applicable to mechanical CAD objects	▪ Primitive shapes for fitting do not cover all the invariances classes in ISO GPS
Spectral Analysis	▪ Use the algebraic properties of its Laplacian	▪ Robust partition regarding deformation	▪ The choice of the type of Laplacian; ▪ the weighting scheme; ▪ the clustering technique
Deep Learning	▪ Address Neural Networks t o process point clouds	▪ Less sensitive to numerical err ors for point attributes	▪ Loss of basic geometrical information; ▪ Ground-truth training data required

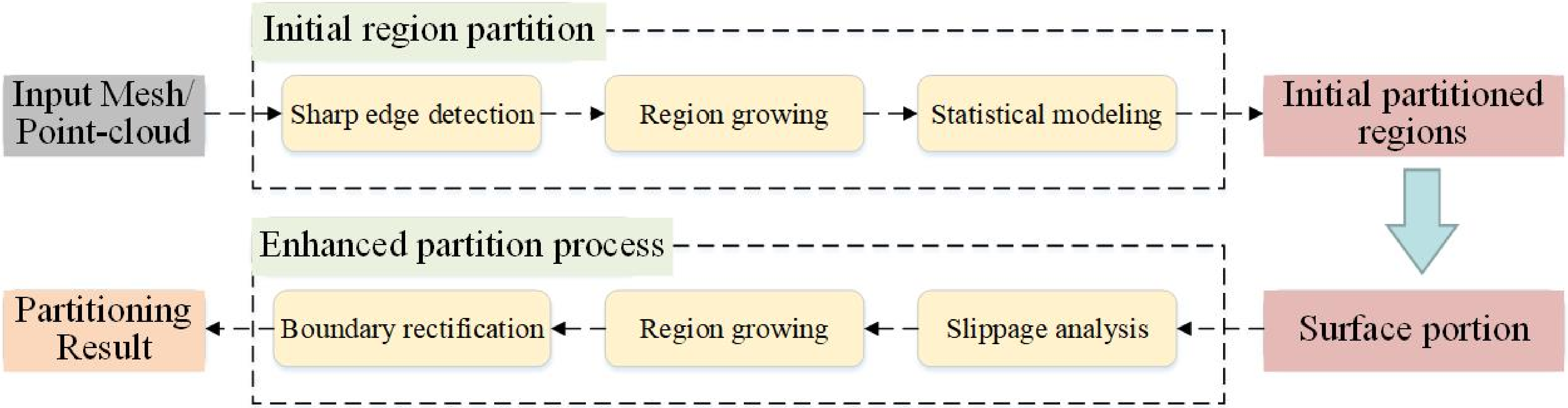
Related work

Observations and Synthesis

- No consideration of invariance classes
- Hybrid methods show better performance
- Most methods rely on discrete curvatures estimation

Method Overview

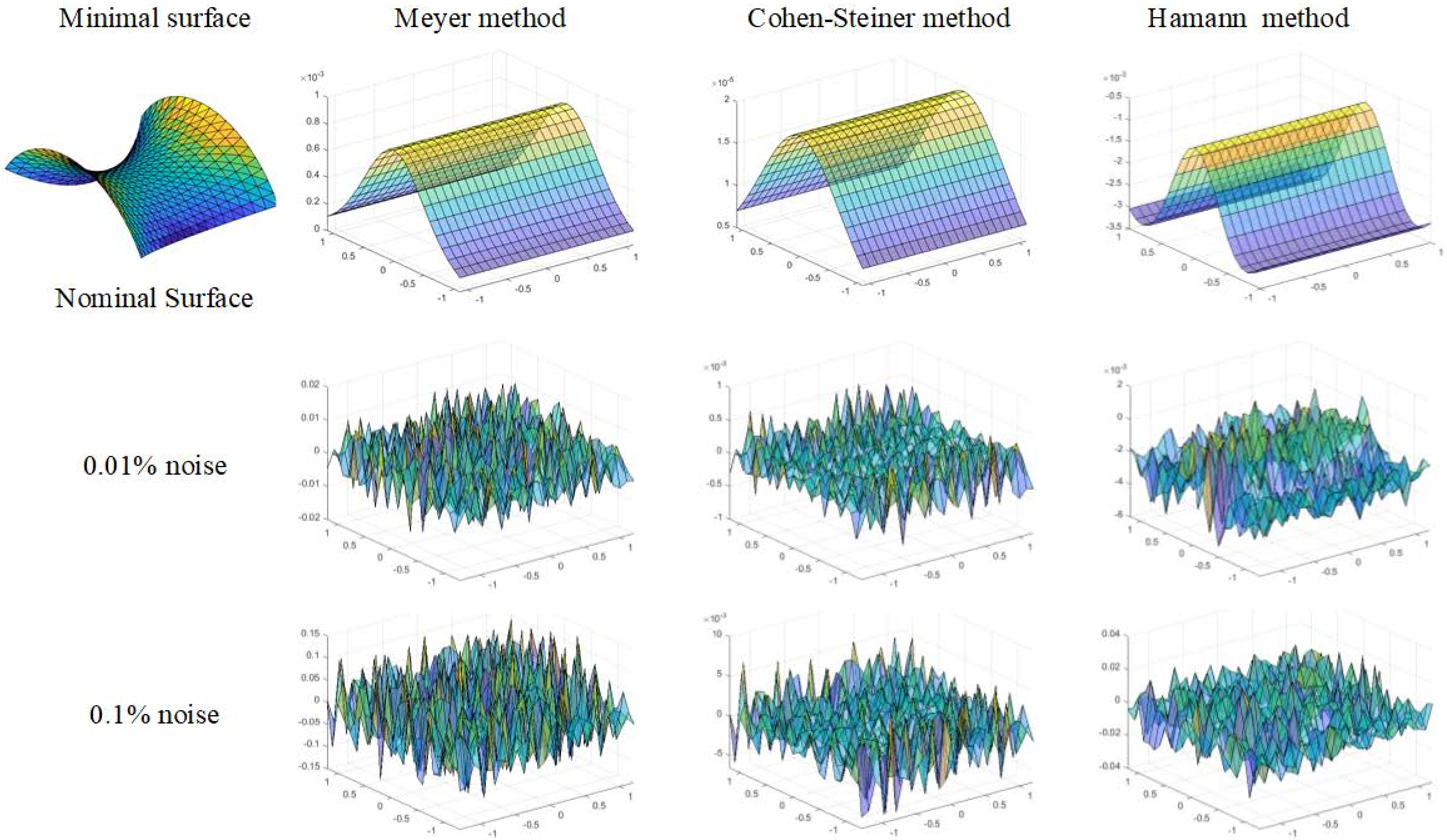
- **The framework of the boundary-based partitioning method**
  - Initial region partition
  - Enhanced partition process



Robust Curvature Estimation

Methods classification

- Discrete differential geometry estimation methods (Meyer. et al.)
- Tensor based methods (Cohen-Steiner et al.)
- Surface fitting methods (Hamann et al.)



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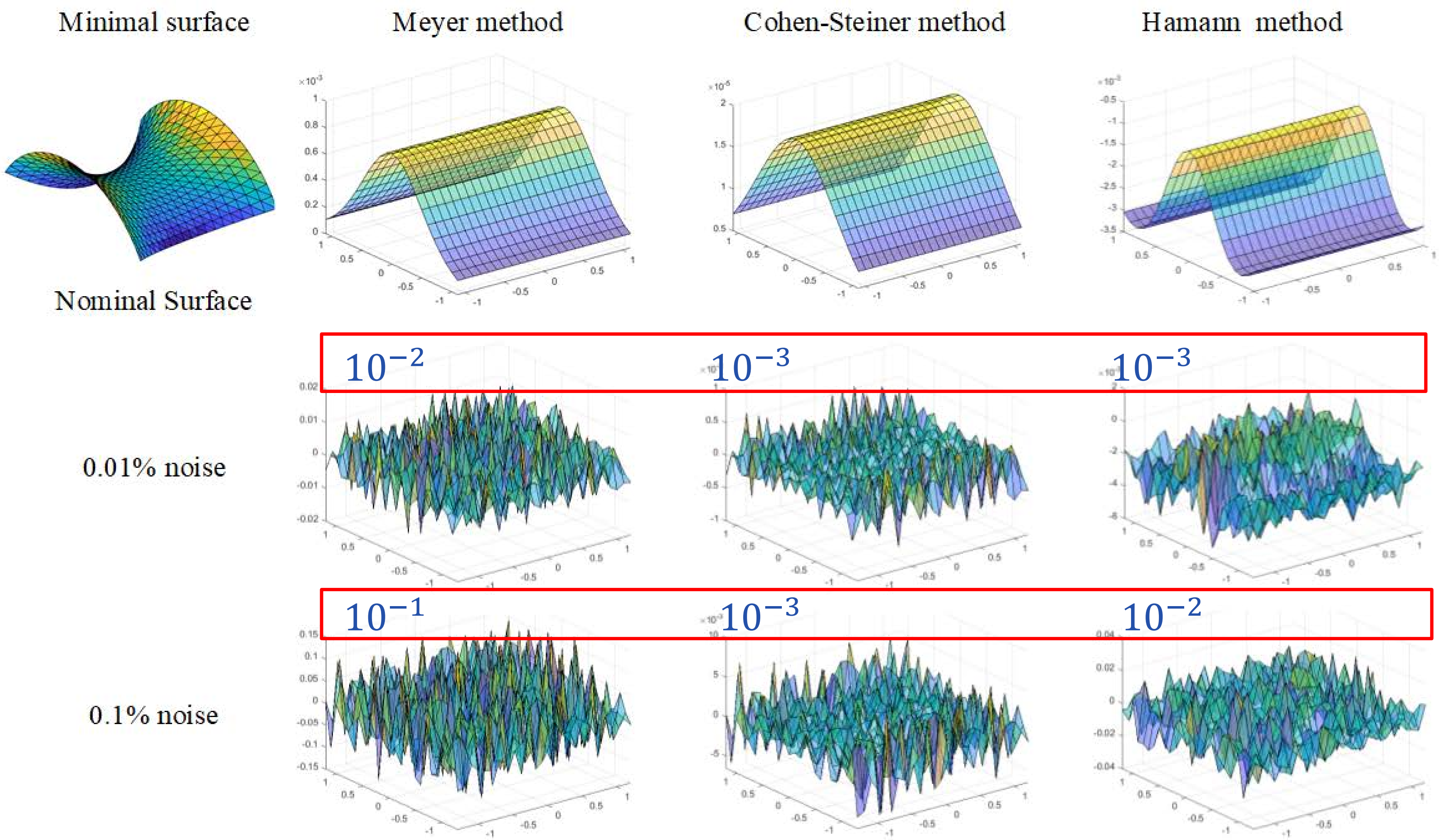

Evaluation of Robust Curvature Estimation

Methods classification

- Discrete differential geometry estimation
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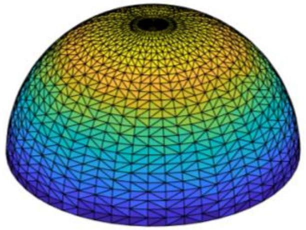
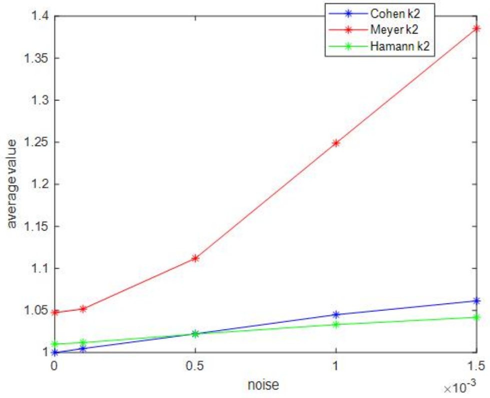
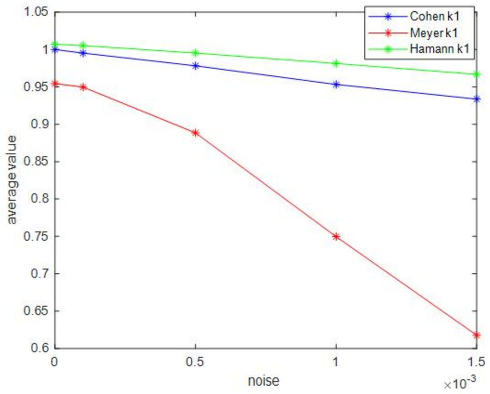
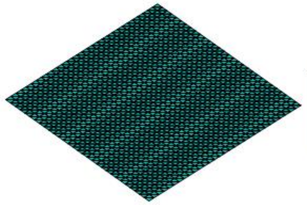
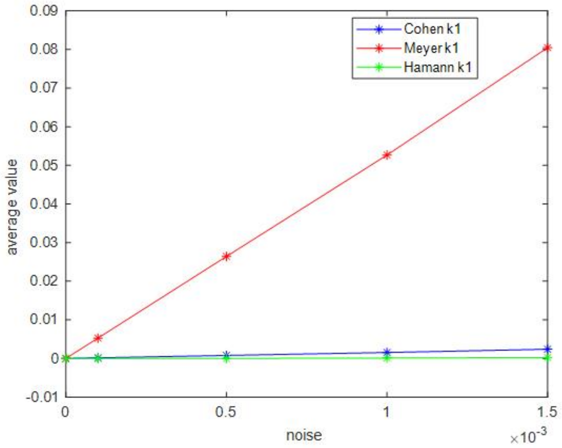
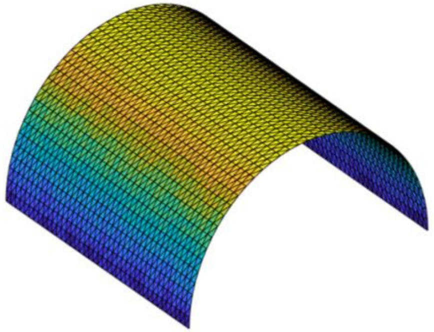


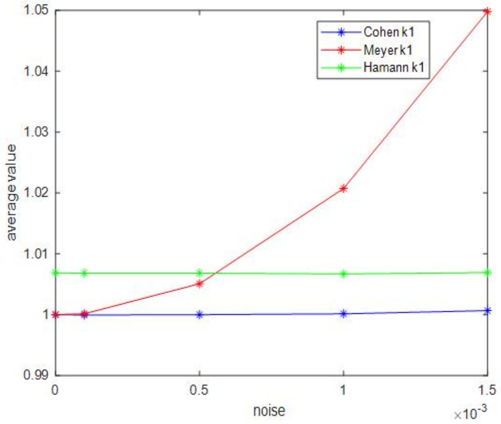


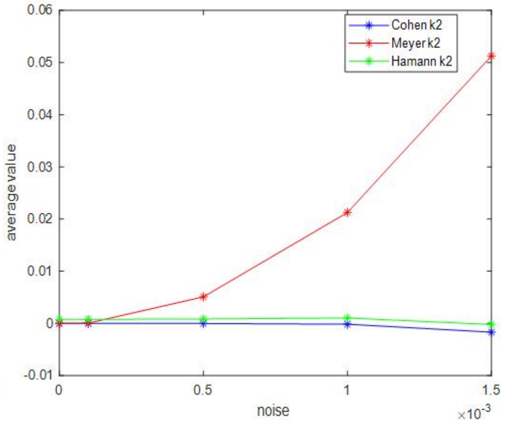












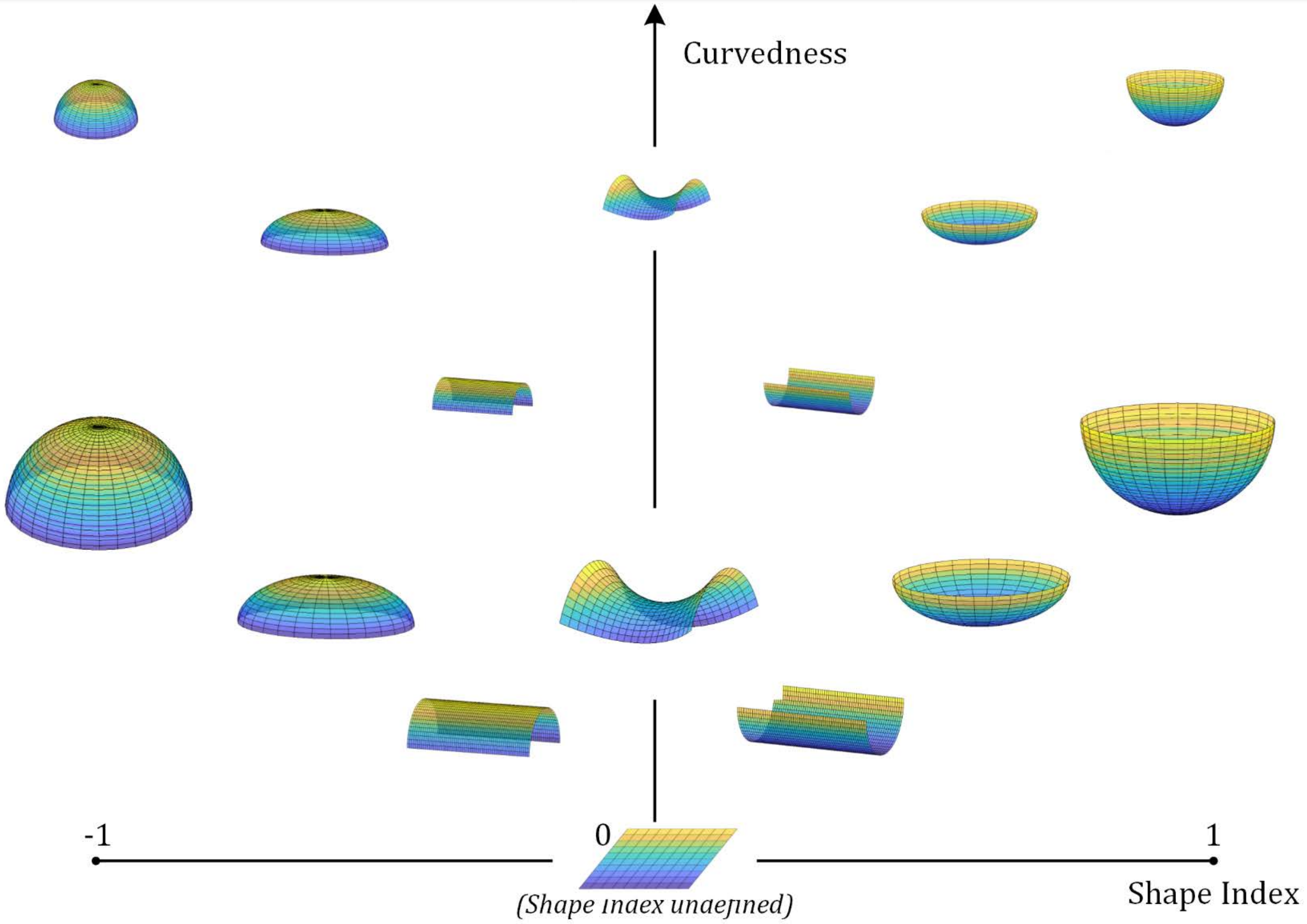
Initial region partition

- **Sharp edge detection**
  - Shape index

$$s(x_i) = \frac{-2}{\pi} arctan\left(\frac{\kappa_1(x_i) + \kappa_2(x_i)}{\kappa_1(x_i) - \kappa_2(x_i)}\right)$$

  - Curvedness

$$c(x_i) = \frac{\sqrt{\kappa_1^2(x_i) + \kappa_2^2(x_i)}}{2}$$



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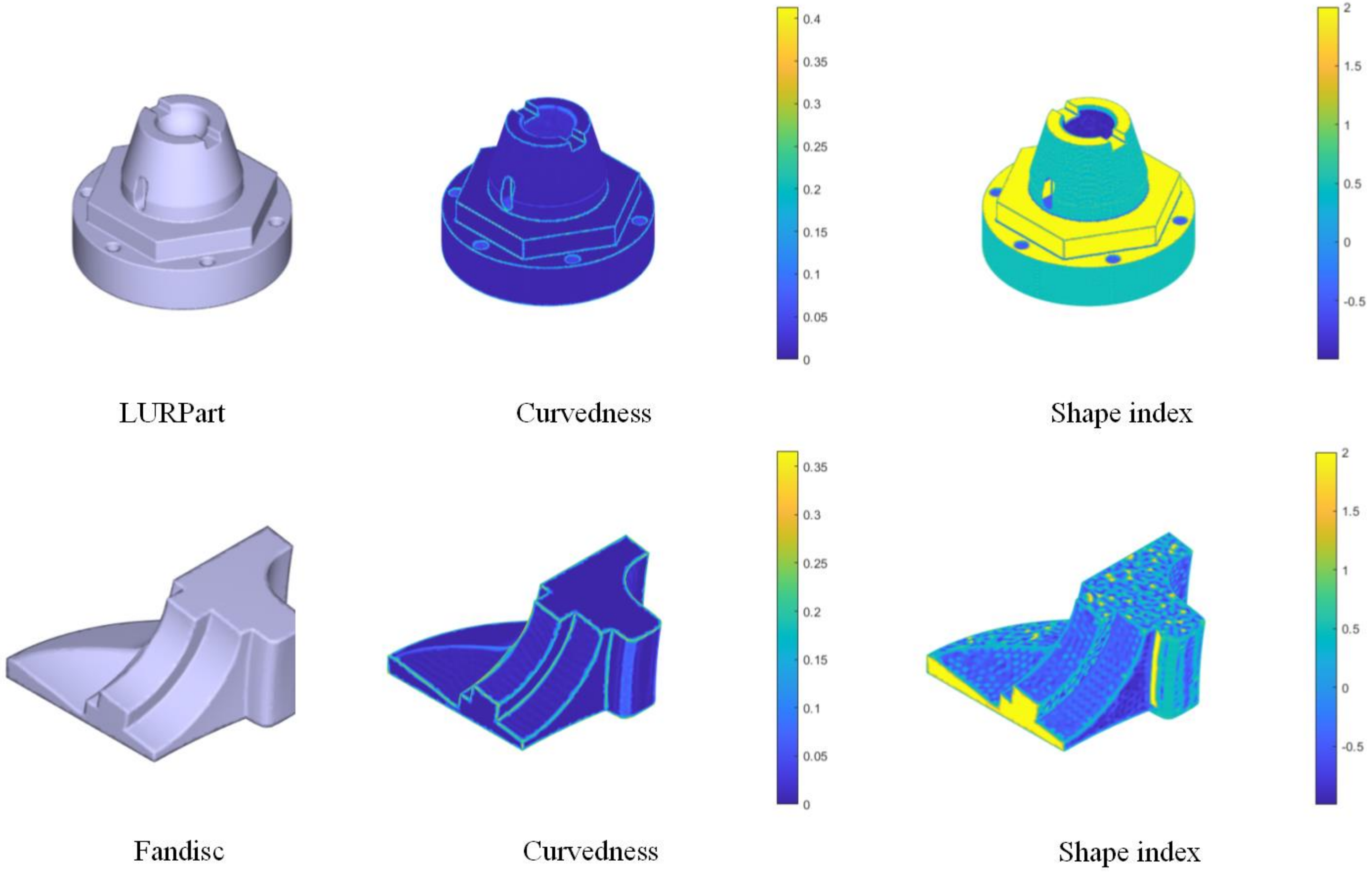

Initial region partition

- **Region growing based on curvedness**
  - c_e is a given threshold to identify sharp edge

$$c_e = \delta \cdot \left(\frac{c_{max} + c_{min}}{2} \right)$$

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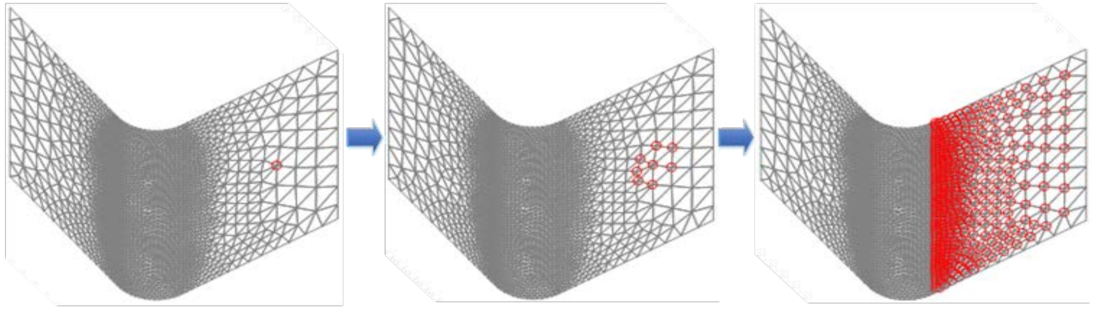

(a) non-shape edge point as the seed

(b) Neighbor points detection

(c) Region growing stops when reaching sharp edge

Region growing process

Initial region partition

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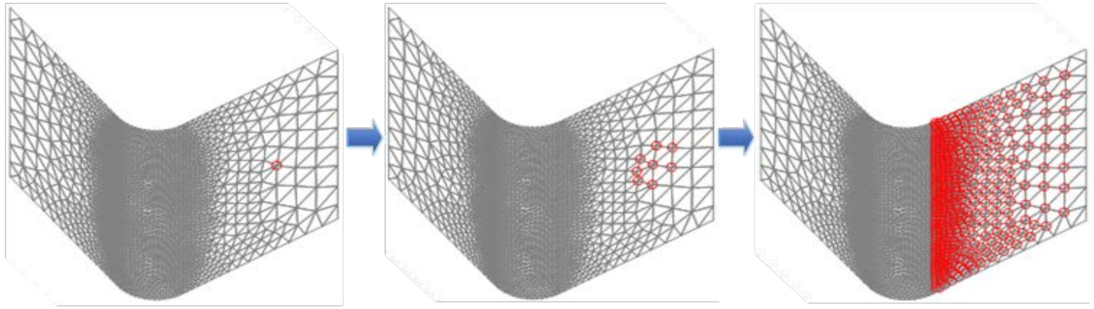

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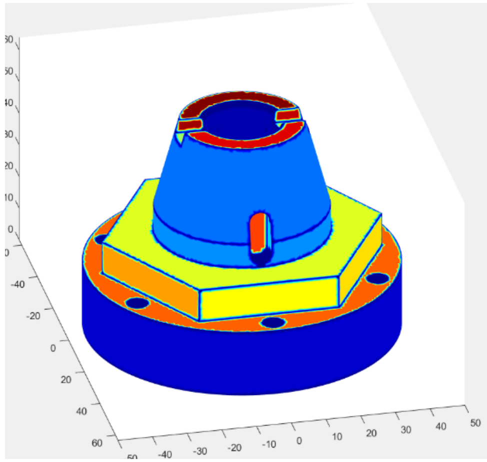
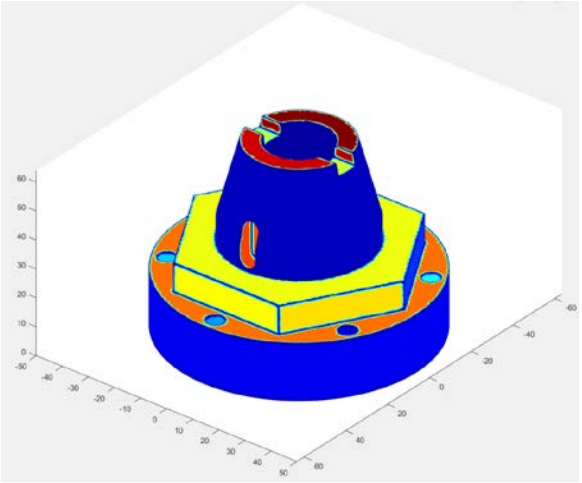
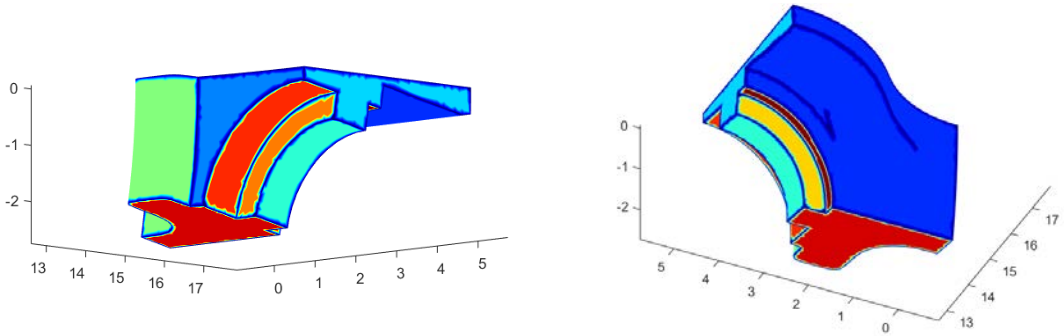


LURPart with $c_e = 0.3$



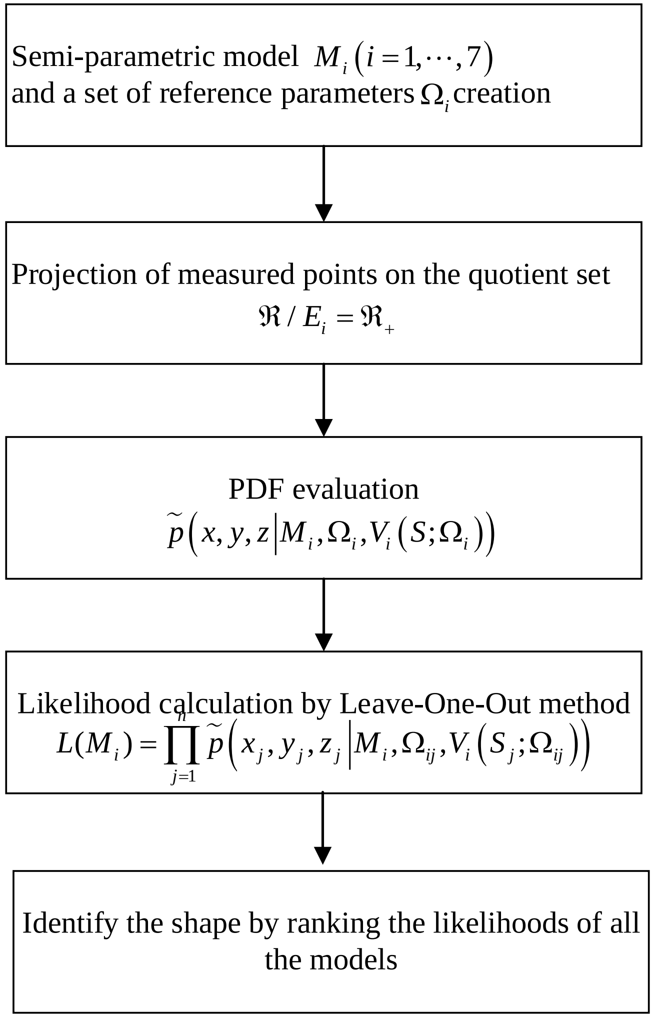
LURPart with $c_e = 0.5$



Fandisc with $c_e = 0.7$

Initial region partition

- **Statistical modeling for invariance class identification** [CC03]

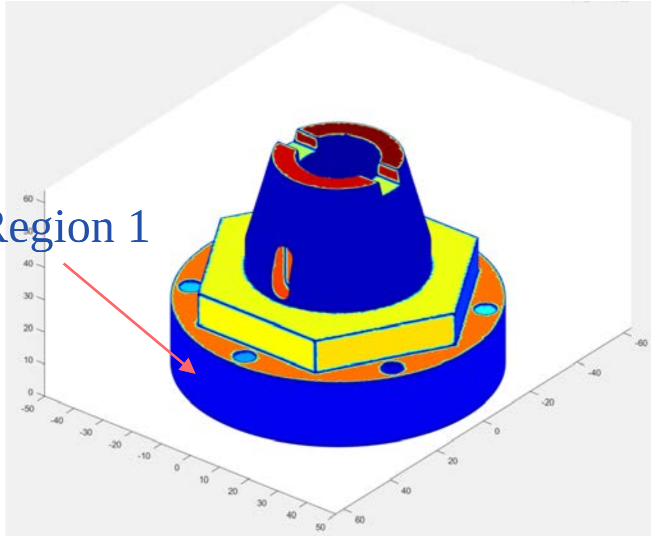


Flowchart of statistical modeling of invariant shapes

Initial region partition

- **Statistical modeling for invariance class identification** [CC03]

Region 1



Invariance class	$\log \hat{L}(M_i)$
Cylindrical	-2.9815 e+03
Sphere	-4.0212 e+03
Revolute	-4.4081 e+03
Prismatic	-4.6734 e+03
Complex	-4.6792 e+03
Planar	-4.9760 e+03

Region 1 and its rankings of against invariance classes

Semi-parametric model $M_i\,(i=1,\cdots,7)$ and a set of reference parameters Ω_i creation

↓

Projection of measured points on the quotient set $\Re / E_i = \Re_+$

↓

PDF evaluation $\tilde{p}\left(x,y,z \mid M_i,\Omega_i,V_i(S;\Omega_i)\right)$

↓

Likelihood calculation by Leave-One-Out method $L(M_i)=\prod_{j=1}^{n}\tilde{p}\left(x_j,y_j,z_j \mid M_i,\Omega_{ij},V_i(S_j;\Omega_{ij})\right)$

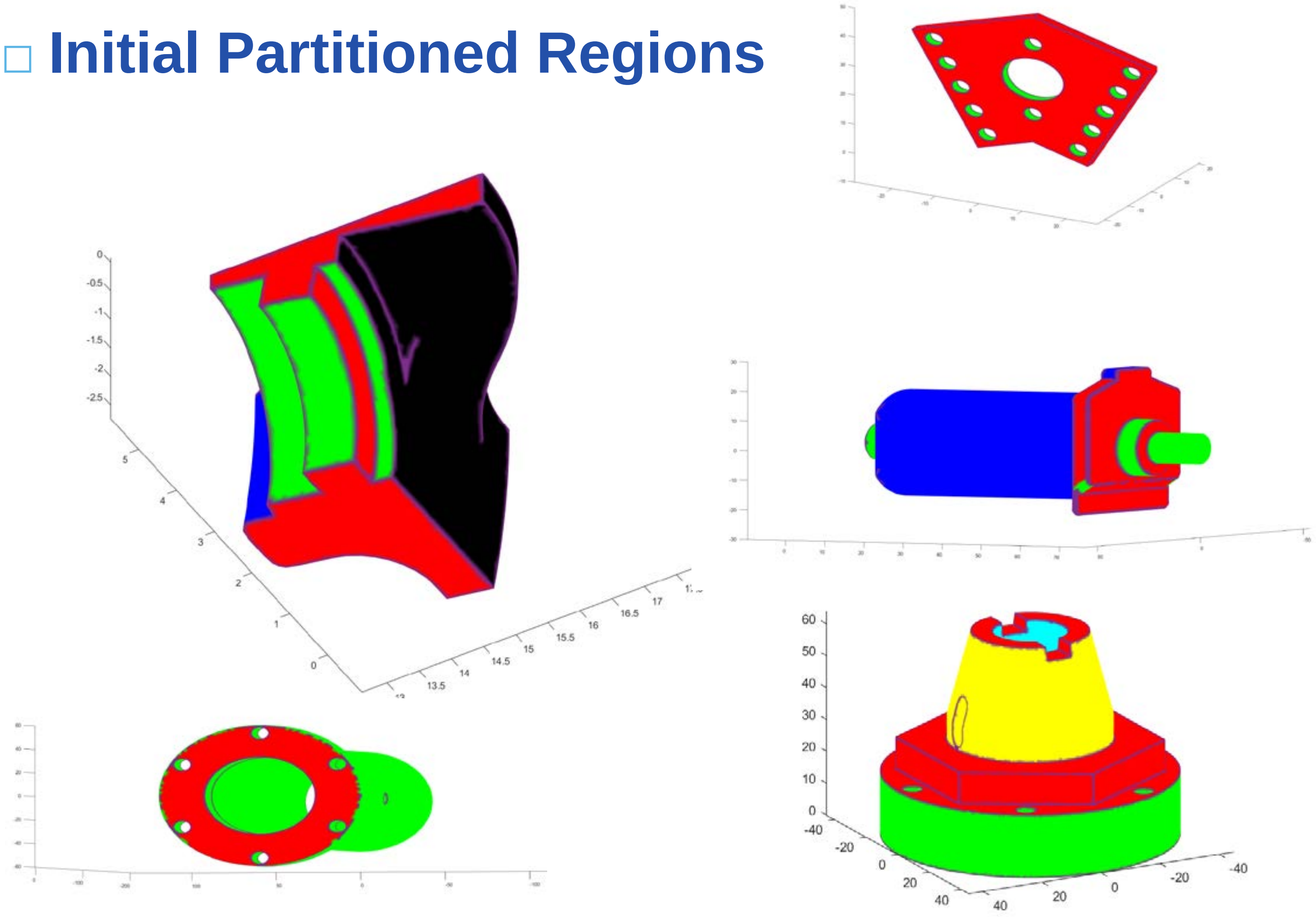
↓

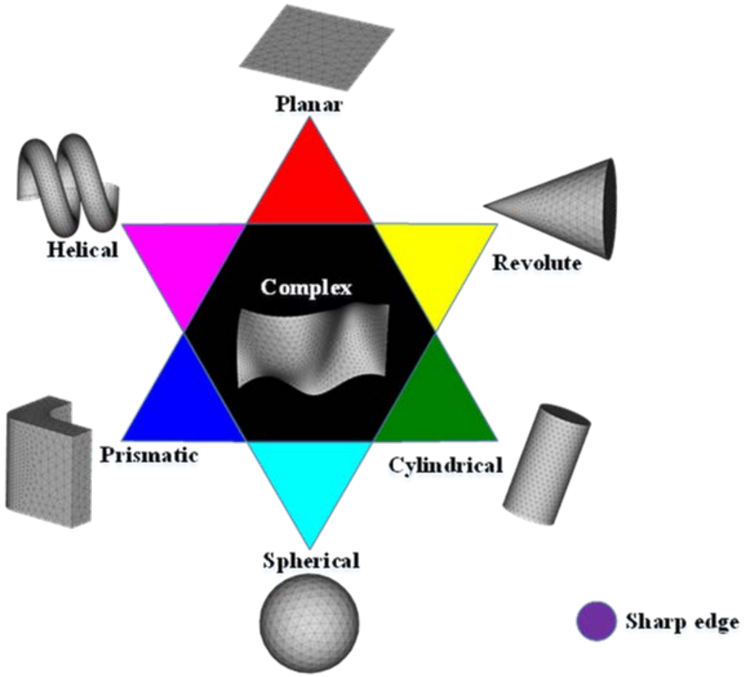
Identify the shape by ranking the likelihoods of all the models

Flowchart of statistical modeling of invariant shapes

Initial region partition

- Initial Partitioned Regions

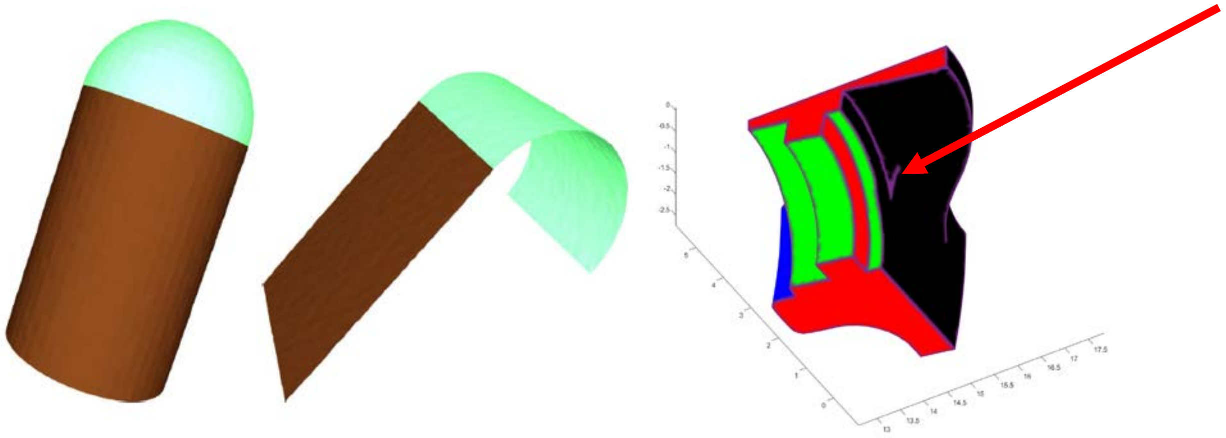




Color Wheel [CAS20]

Enhanced partitioning process

- **Objective**
  - According to ISO GPS, connected regions could have natural boundaries that contains no abrupt change of point differential properties [Feature Principle – ISO 8015:2011].



Examples of boundaries without sharp edge

Enhanced partitioning process

Region growing by slippage analysis

- Slippage analysis [CG04]
  - Rigid motion: $\mathbf{x}(t) = R(t) \cdot \mathbf{x}_0 + T(t)$ Instant velocity: $\mathbf{v}(\mathbf{x}) = \mathbf{r} \times \mathbf{x}_0 + \mathbf{t}$
  - If the instant velocity vector of each point $\mathbf{x} \in S$ is tangent to the surface S under rigid motion M, then we call M as **slippable motion** and the surface S is a **kinematic surface**.

$$\max_{[r\ t]} \sum_{i=1}^{n} ((\boldsymbol{r} \times \boldsymbol{x}_i + \boldsymbol{t}) \cdot \boldsymbol{n}_i)^2$$

Enhanced partitioning process

□ Region growing by slippage analysis

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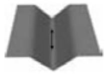
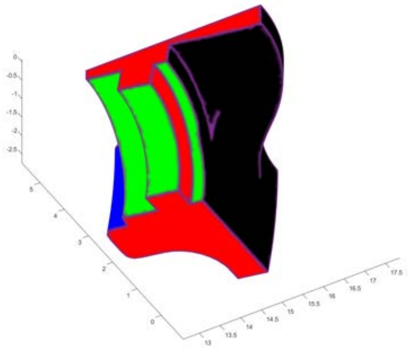
$$\max_{[r\ t]} \sum_{i=1}^{n} ((\boldsymbol{r} \times \boldsymbol{x}_i + \boldsymbol{t}) \cdot \boldsymbol{n}_i)^2$$

$$M_c = \sum_{i=1}^{n} \begin{bmatrix} c_{ix}c_{ix} & c_{ix}c_{iy} & c_{ix}c_{iz} & c_{ix}n_{ix} & c_{ix}n_{iy} & c_{ix}n_{iz} \\ c_{iy}c_{ix} & c_{iy}c_{iy} & c_{iy}c_{iz} & c_{iy}n_{ix} & c_{iy}n_{iy} & c_{iy}n_{iz} \\ c_{iz}c_{ix} & c_{iz}c_{iy} & c_{iz}c_{iz} & c_{iz}n_{ix} & c_{iz}n_{iy} & c_{iz}n_{iz} \\ n_{ix}c_{ix} & n_{ix}c_{iy} & n_{ix}c_{iz} & n_{ix}n_{ix} & n_{ix}n_{iy} & n_{ix}n_{iz} \\ n_{iy}c_{ix} & n_{iy}c_{iy} & n_{iy}c_{iz} & n_{iy}n_{ix} & n_{iy}n_{iy} & n_{iy}n_{iz} \\ n_{iz}c_{ix} & n_{iz}c_{iy} & n_{iz}c_{iz} & n_{iz}n_{ix} & n_{iz}n_{iy} & n_{iz}n_{iz} \end{bmatrix}$$

Num. small eigenvalues	Type of eigenvectors	Type of Surface	
3	3 rotations	sphere	
3	2 translation, 1 rotation	plane	
2	1 translation, 1 rotation	cylinder	
1	translation	linear extrusion	
1	rotation	surface of revolution	
1	helical motion	helix	

Enhanced partitioning process

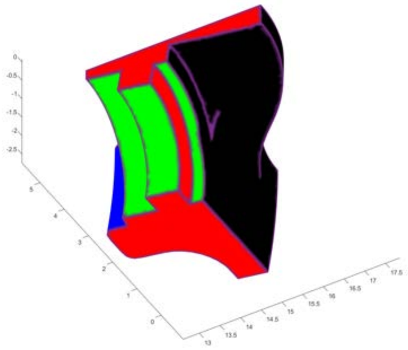
- Region growing by slippage analysis



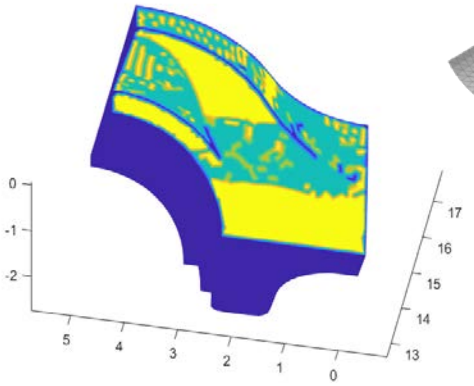
(a) Initial Region Segmentation
(Complex surface in black)

Enhanced partitioning process

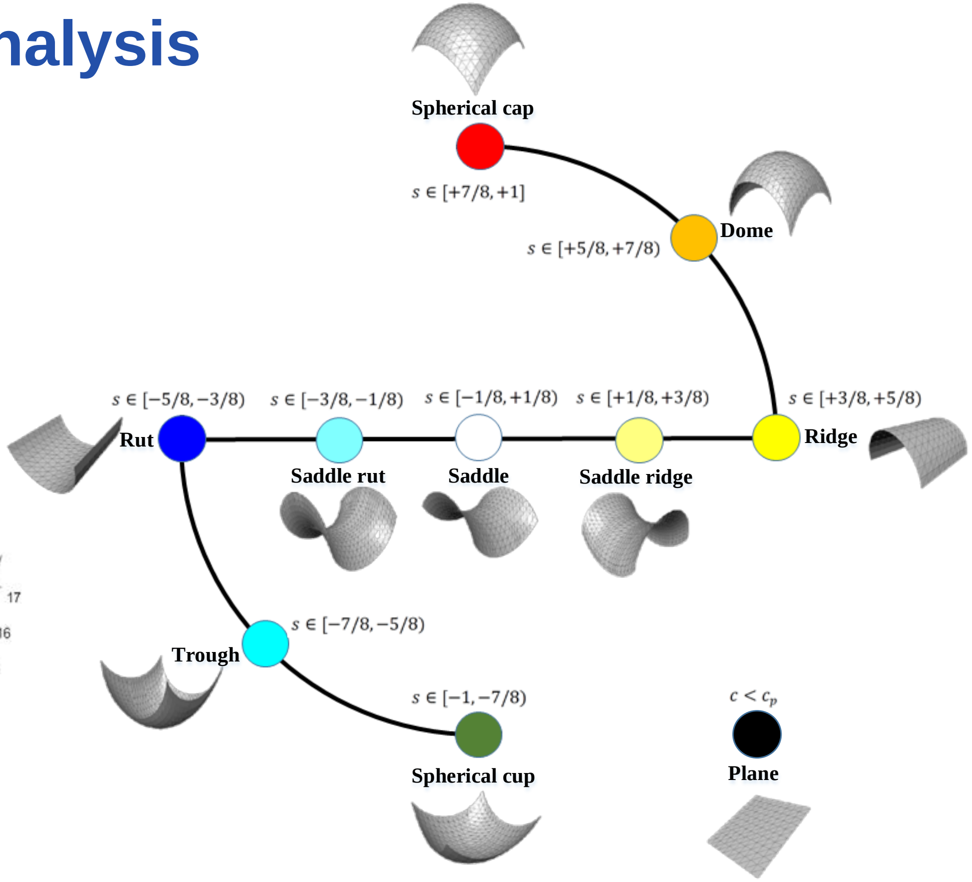
□ Region growing by slippage analysis



(a) Initial Region Segmentation
(Complex surface in black)



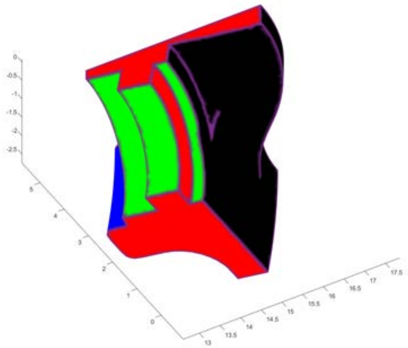
(b) Shape index



(c) Grouping vertices by
shape index

Enhanced partitioning process

- Region growing by slippage analysis



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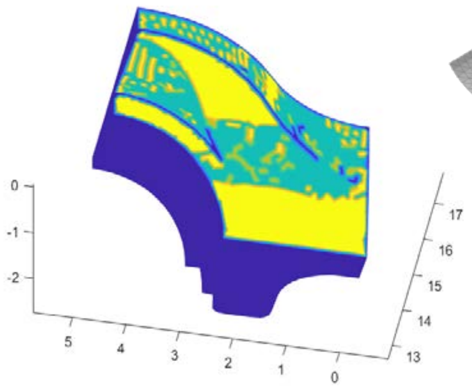
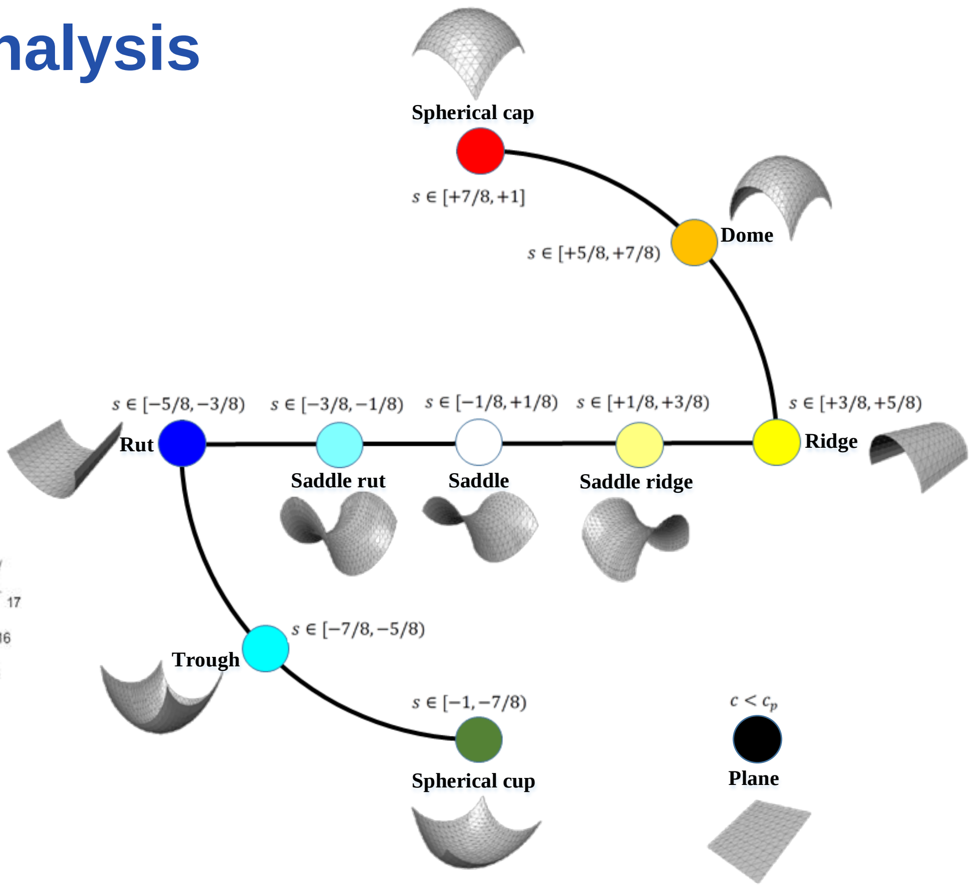
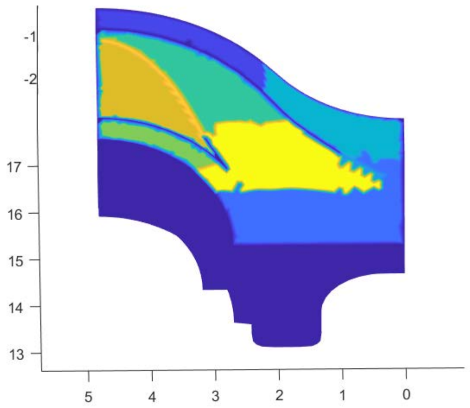


(b) Shape index



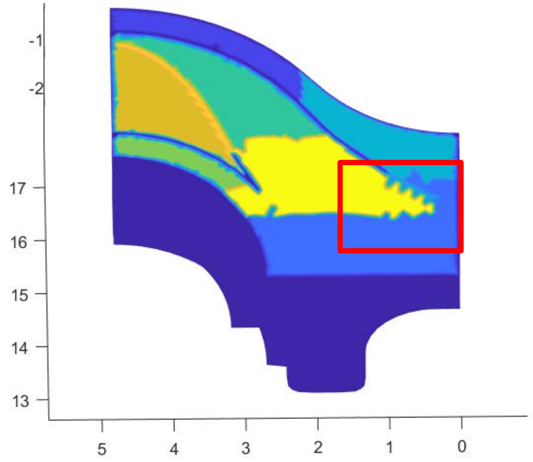
(c) Grouping vertices by shape index



(d) Merging neighbor groups by slippable motions

Enhanced partitioning process

- **Boundary rectification based on Conformal Geometry**
  - Conformal Geometry
    - A map $f : M \to N$ between two Riemann surfaces is called to be **conformal** if there exists a positive scalar function λ such that $f^{*}ds_N^2 = \lambda ds_M^2$.
    - Conformal mapping **preserves topology and angles**.
    - All closed surfaces can be conformally deformed to one of the three canonical spaces: the unit sphere, the plane or the hyperbolic space.



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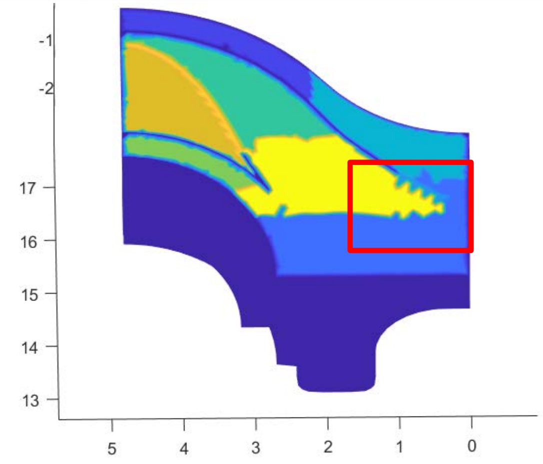
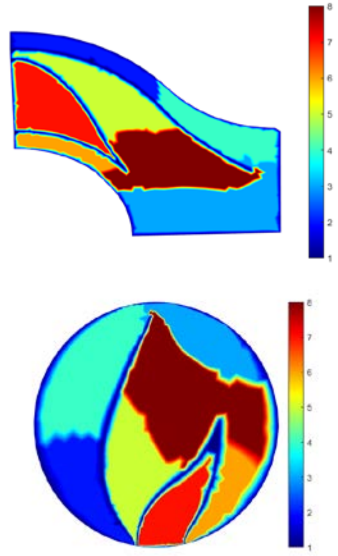


Map on a Sphere

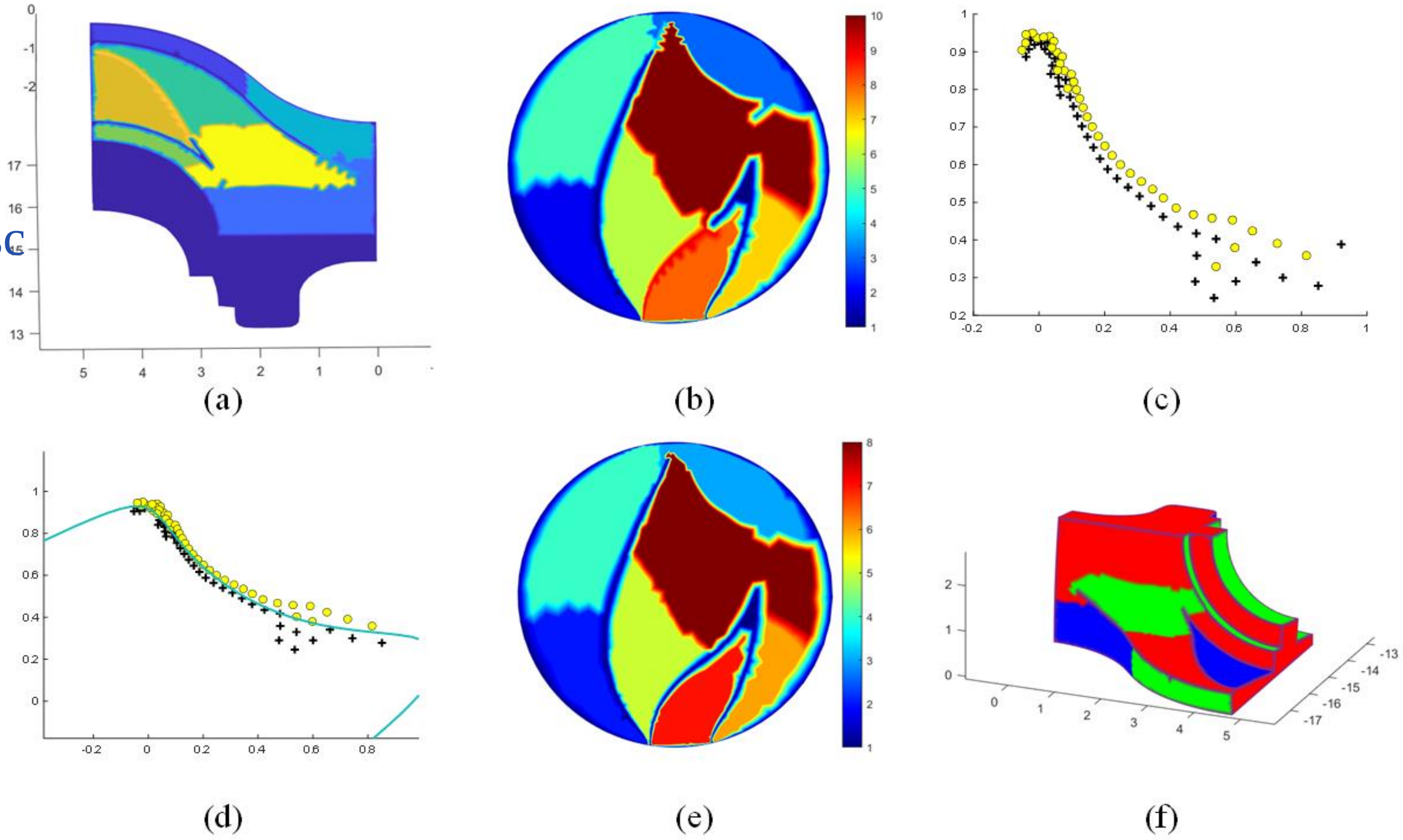


Map on a unit Disc

Enhanced partitioning process

□ Boundary rectification pipeline

- (a) region growing result based on slippage analysis
- (b) conformal mapping onto a unit disc
- (c) boundary points detection on the disc
- (d) boundary rectification by logistic regression
- (e)refined surface re-mapping back to the part
- (f) surface identification by statistical modeling



Enhanced partitioning process

Boundary rectification pipeline

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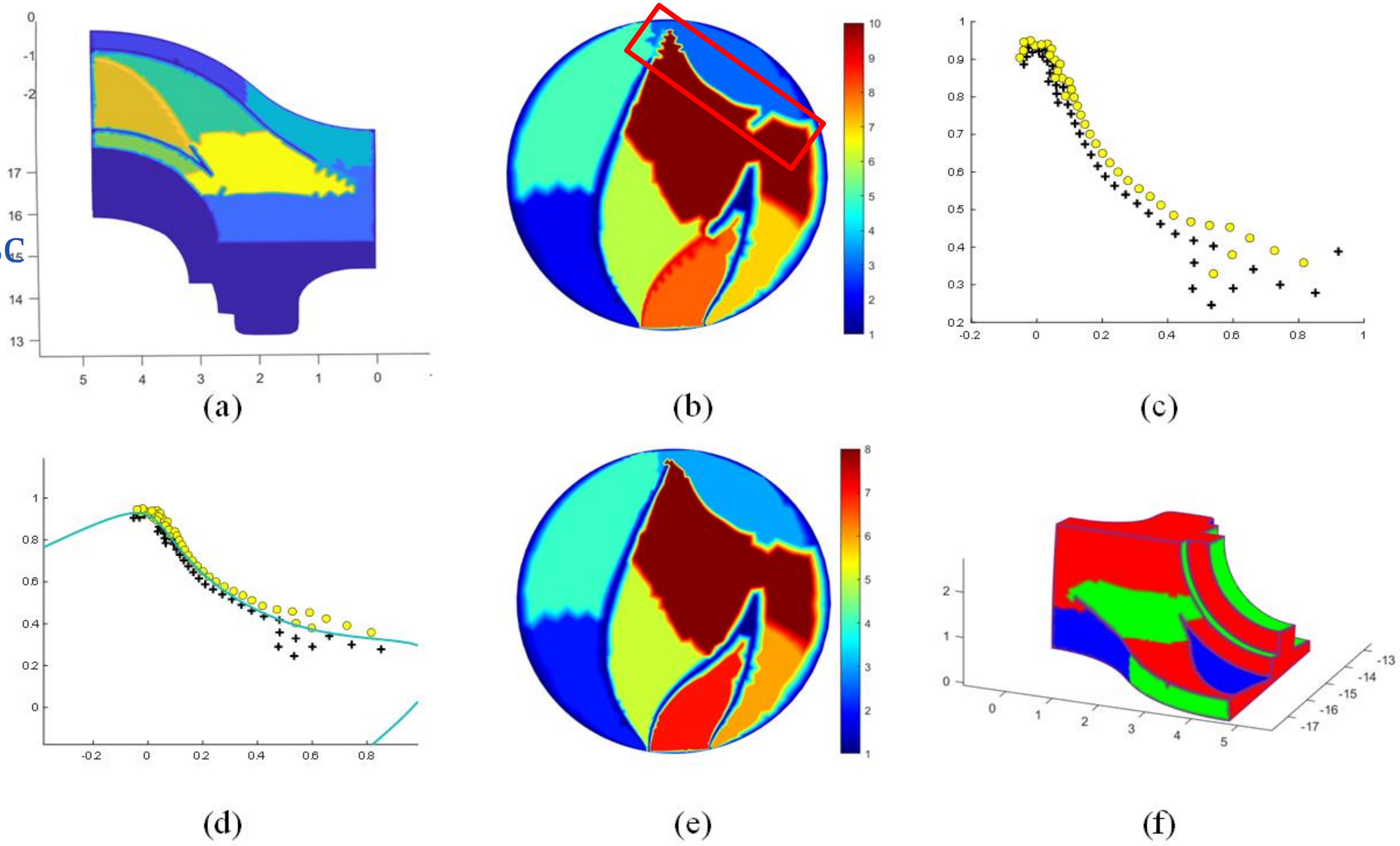
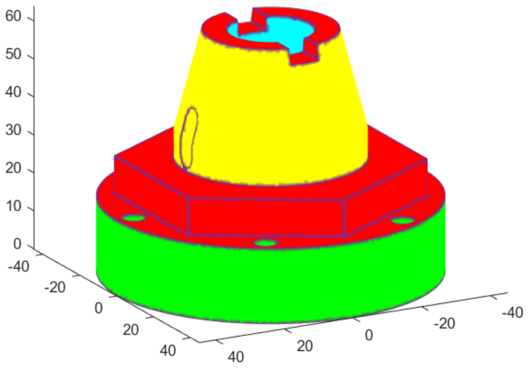
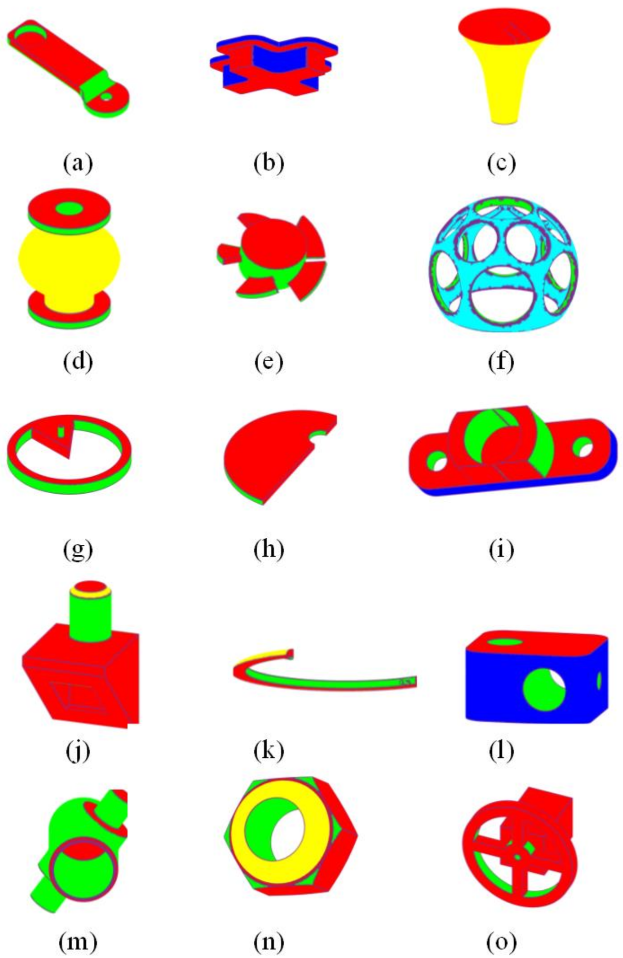


Case studies

- Robust results
  - Boundaries
  - Noise
  - Data density
  - Sampling distribution



33827 vertices 179s



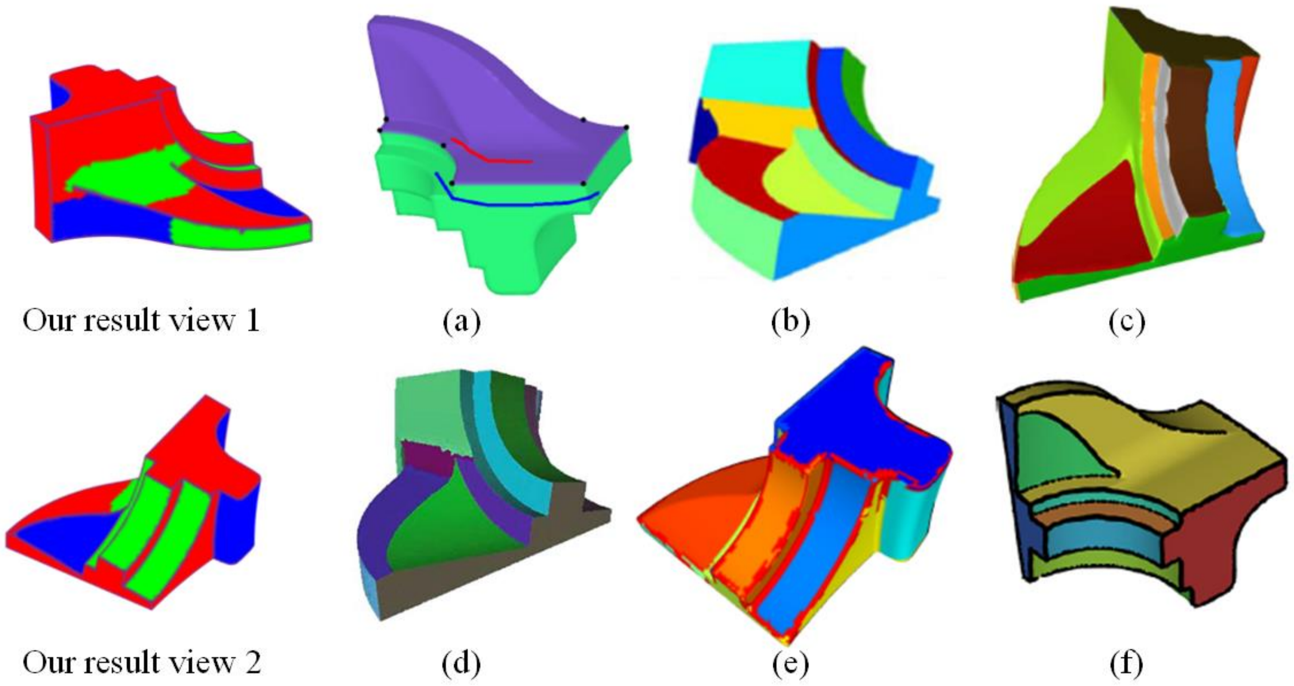
Part	No. vertices	No. features	Cal. time
a	8681	15	94s
b	14333	15	123s
c	42785	5	1593s
d	22756	8	196s
e	13440	33	125s
f	15269	16	157s
g	7652	8	35s
h	9325	6	80s
i	19478	14	105s
j	18775	14	189s
k	2094	9	15s
l	35462	8	911s
m	20621	13	336s
n	21613	21	318s
o	20339	27	192s

Testing result of the method[parts: RMP19]

Case studies

□ Comparison with existing methods

- Invariance class
- Automatic
- Non-sharp edge
- Robust



The partition results of Fandisc part by (a) interactive edge detection method [ZZC10], no final result is provided by the authors; (b) region growing method [LDB04]; (c) hierarchical clustering method [LHM08]; (d) shape fitting method [AFS06] (e) spectral analysis method [WI18] (f) deep learning method [RMP19]

Conclusion and Outlook

- Draw a classification of segmentation methods
- Evaluate curvature estimation methods
- Propose a boundary-based mesh partitioning method regarding ISO GPS
- Conformal geometry is used to map a 3D surface onto a 2D unit disc

- Improve vertex clustering methods considering non-default partitioning
- Test on shape with complex topologies
- Validate on measured parts
- Evaluate of partitioning results

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Thanks For Your Attention